Outline

- What is part of speech tagging?
- Markov chains
- Hidden Markov models
- Viterbi algorithm
- Example
- Coding assignment!

What is part of speech?

Why	not	learn	something	?
adverb	adverb	verb	noun	punctuation mark, sentence closer

Part of speech (POS) tagging

Part of speech tags:

lexical term	tag	example
noun	NN	something, nothing
verb	VB	learn, study
determiner	DT	the, a
w-adverb	WRB	why, where
...	...	

Part of speech (POS) tagging

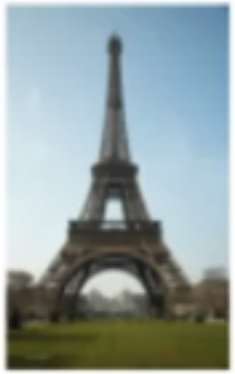
Part of speech tags:

lexical term	tag	example
noun	NN	something, nothing
verb	VB	learn, study
determiner	DT	the, a
w-adverb	WRB	why, where
...	...	

Why not learn something ?

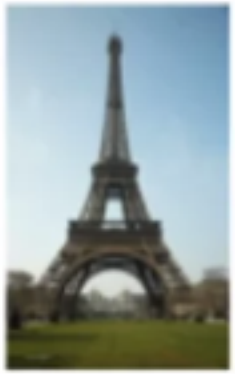
WRB RB VB NN .

Applications of POS tagging

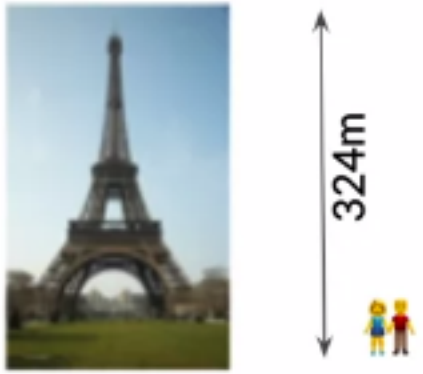


Named entities

Applications of POS tagging

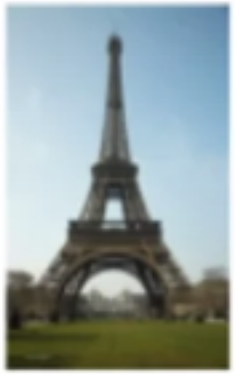


Named entities

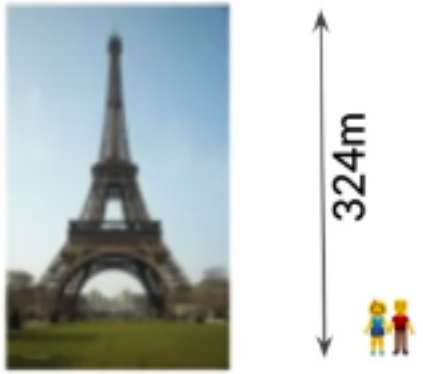


Co-reference resolution

Applications of POS tagging



Named entities



Co-reference resolution

Speech recognition

Example

Why not learn ...

verb

Example

Why not learn ...

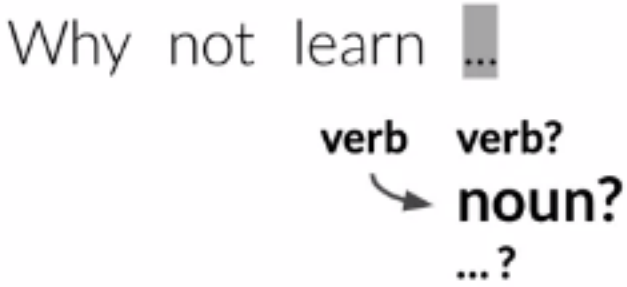
verb **verb?**
noun?
... ?

Part of Speech Dependencies

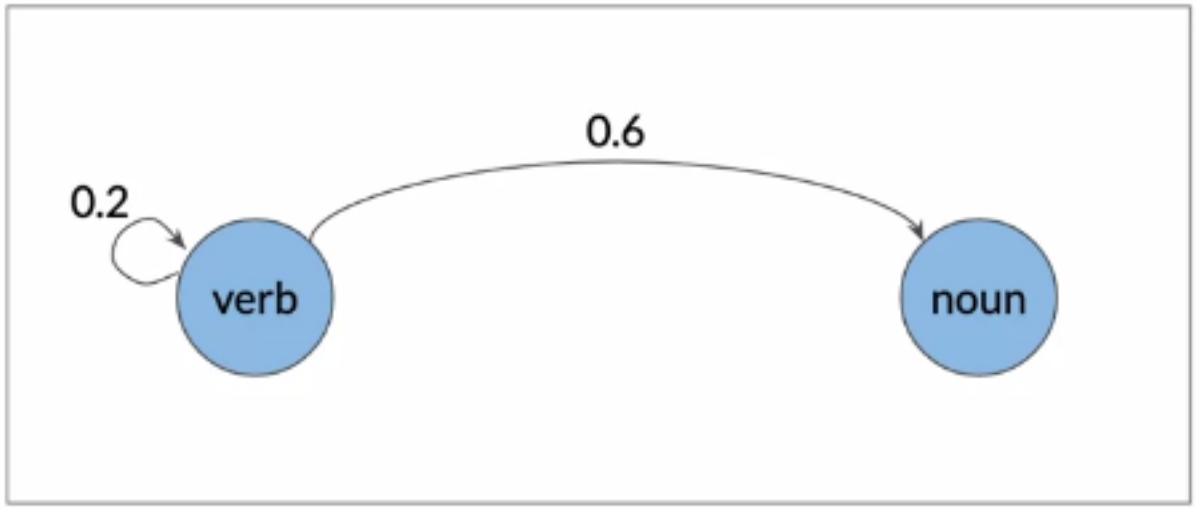
Why not learn ...

verb verb?
noun?
...?

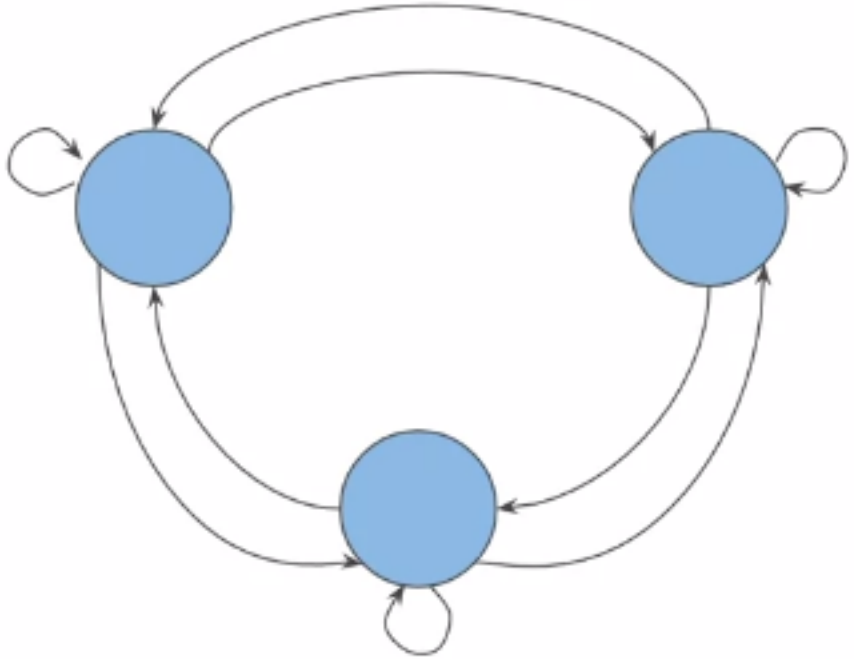
Part of Speech Dependencies



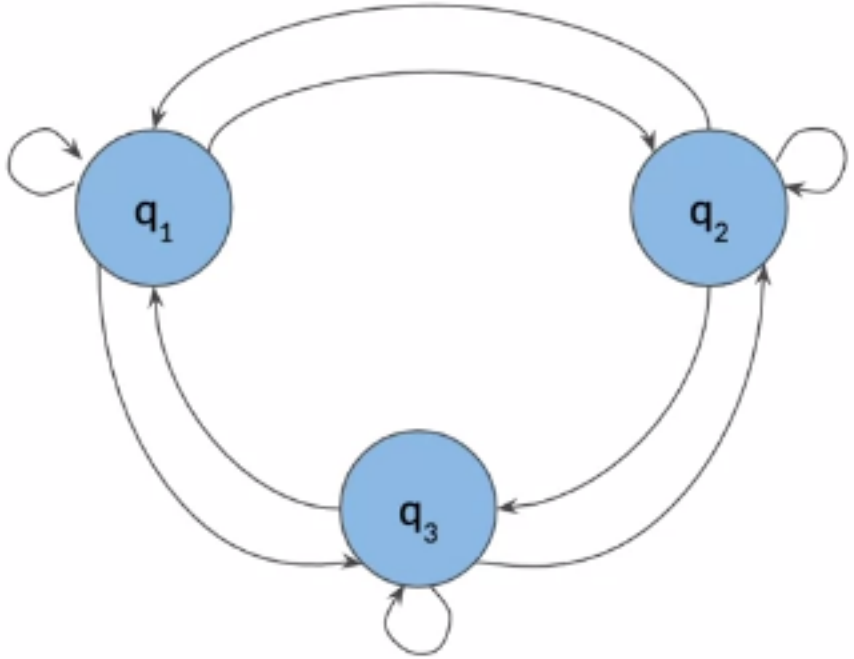
Visual Representation



What are Markov chains?



States



$$Q = \{q_1, q_2, q_3\}$$

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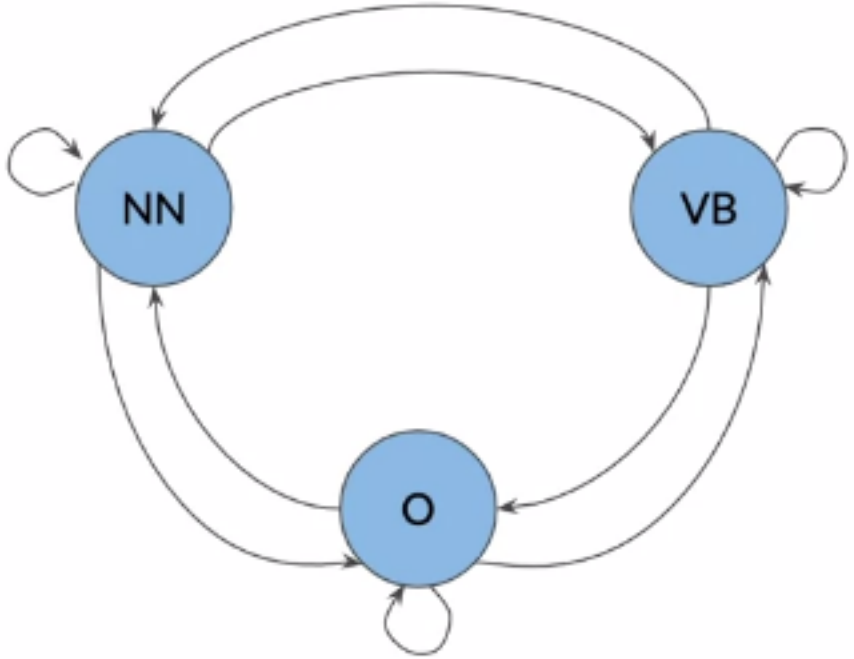
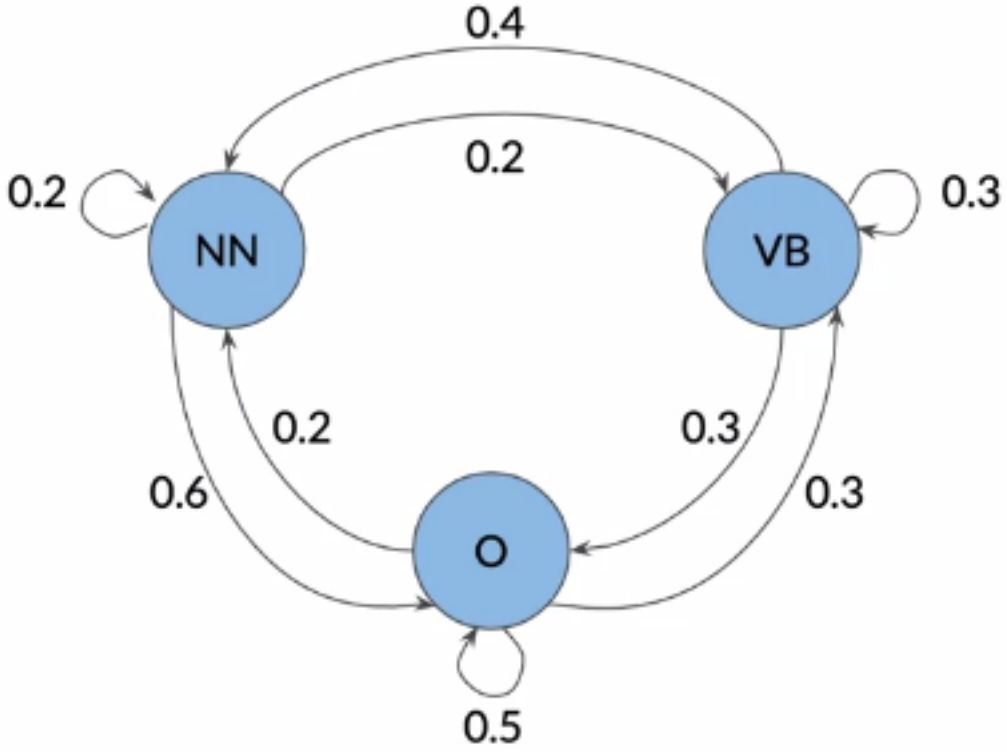
Markov Chains and POS Tags

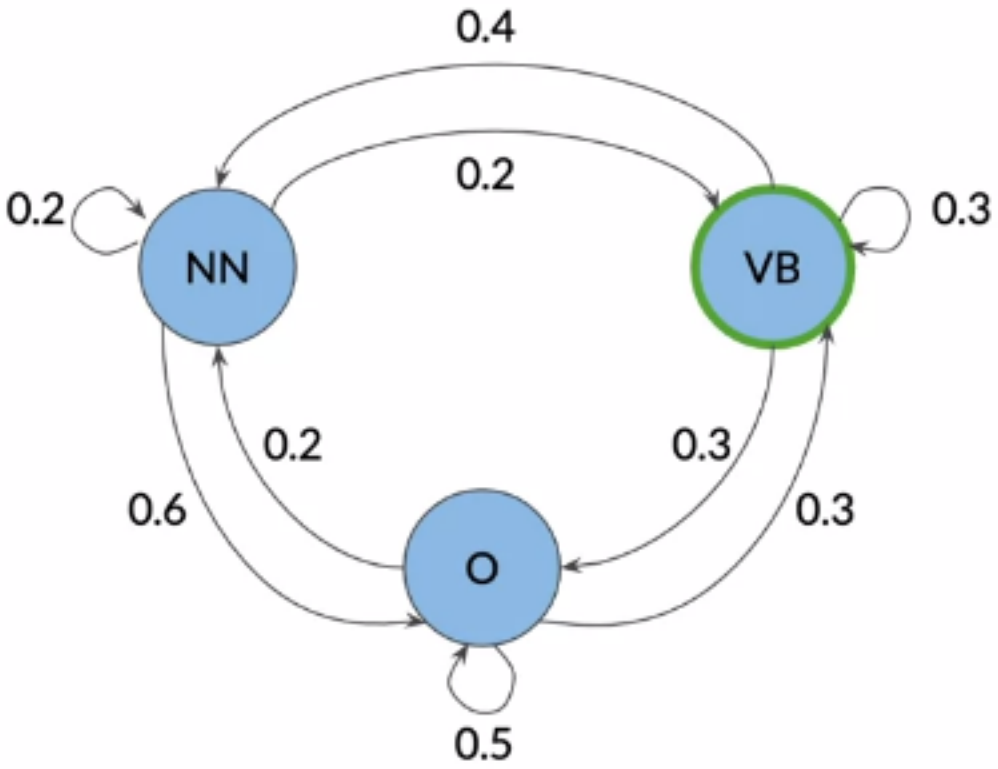
POS tags as States



Transition probabilities

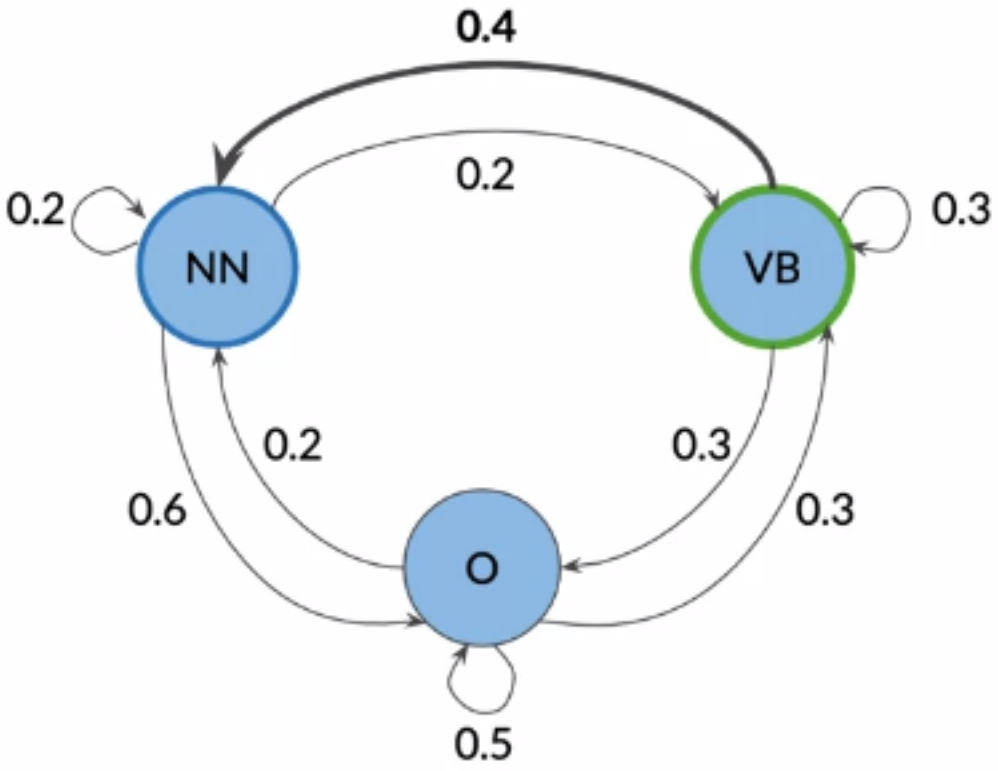


Transition probabilities



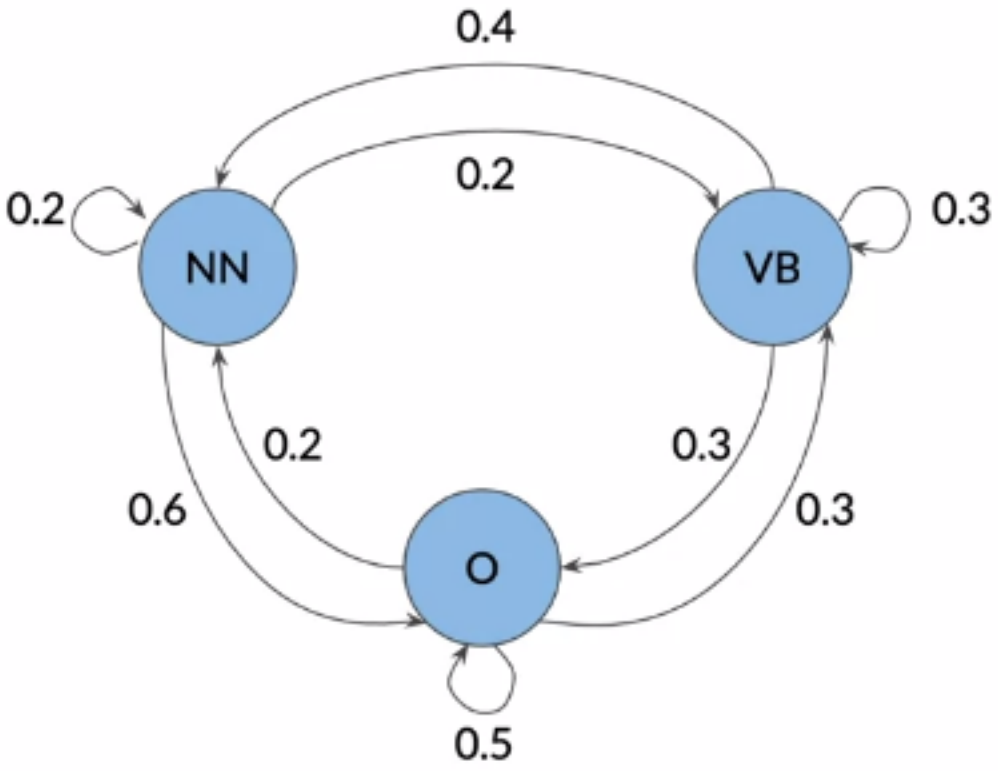
Why not learn something ?

Transition probabilities



Why not learn something ?

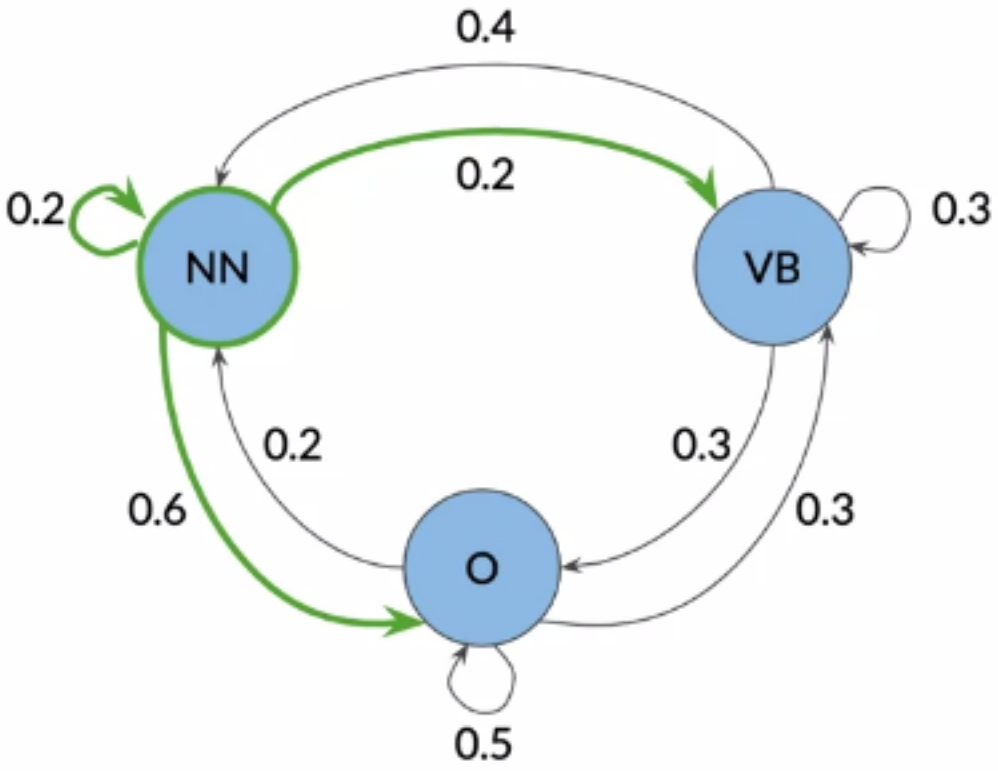
The transition matrix



$A =$

	NN	VB	O
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

The transition matrix



$A =$

	NN	VB	O
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
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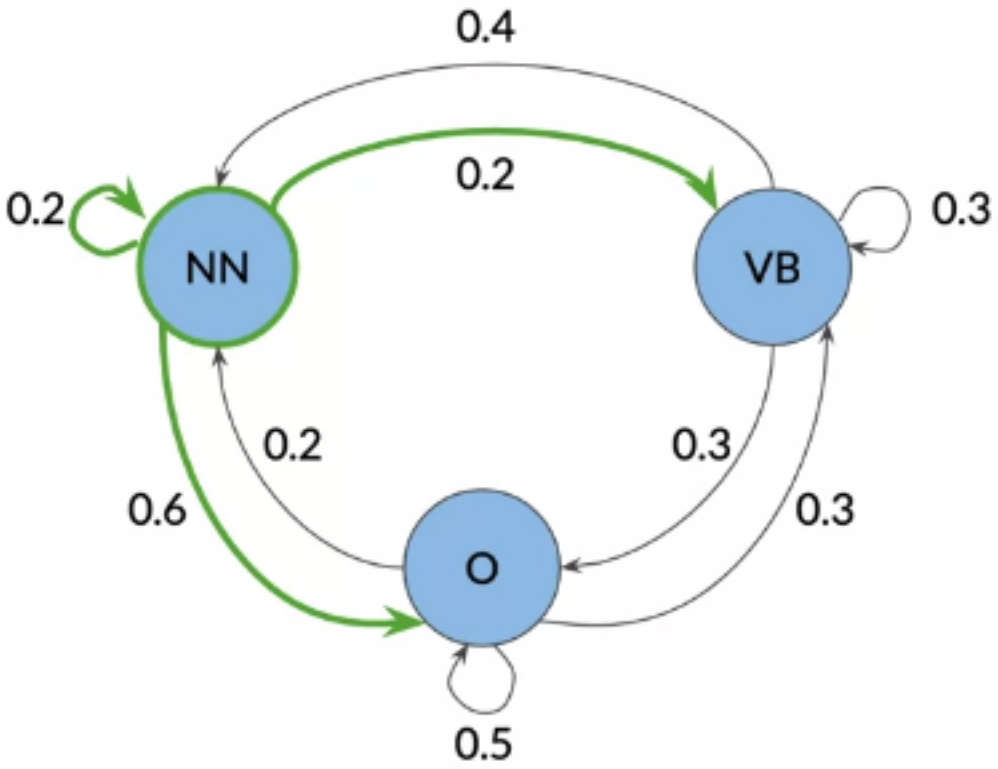
The transition matrix

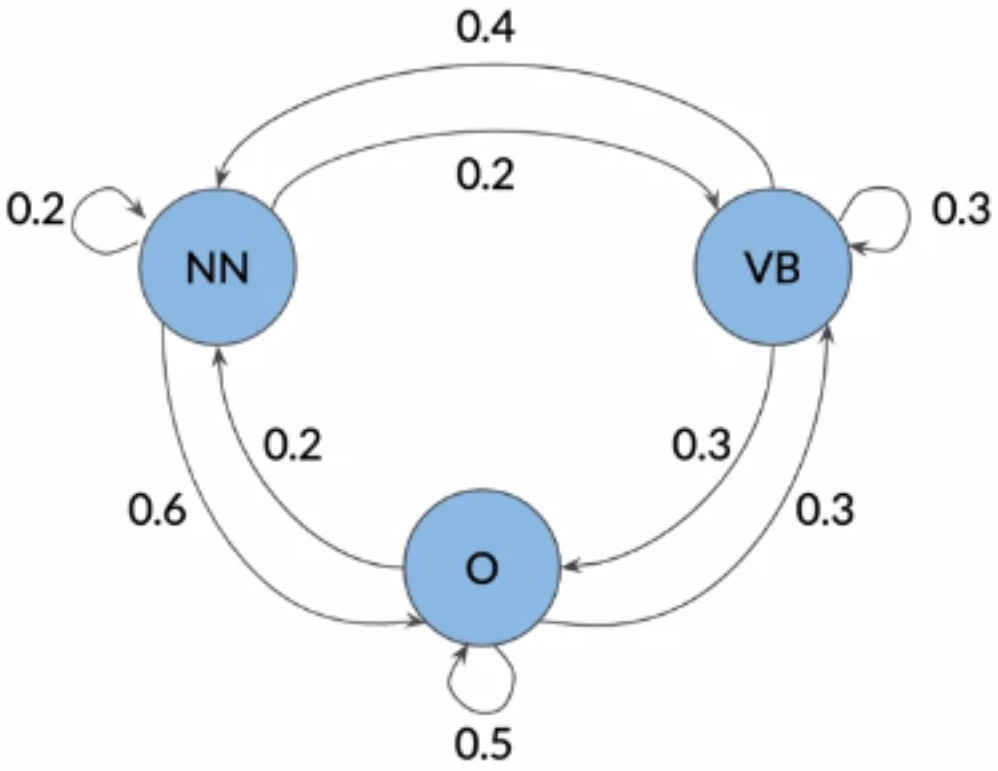


$$A =$$

	NN	VB	O
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

$$\sum_{j=1}^{N} a_{ij} = 1$$

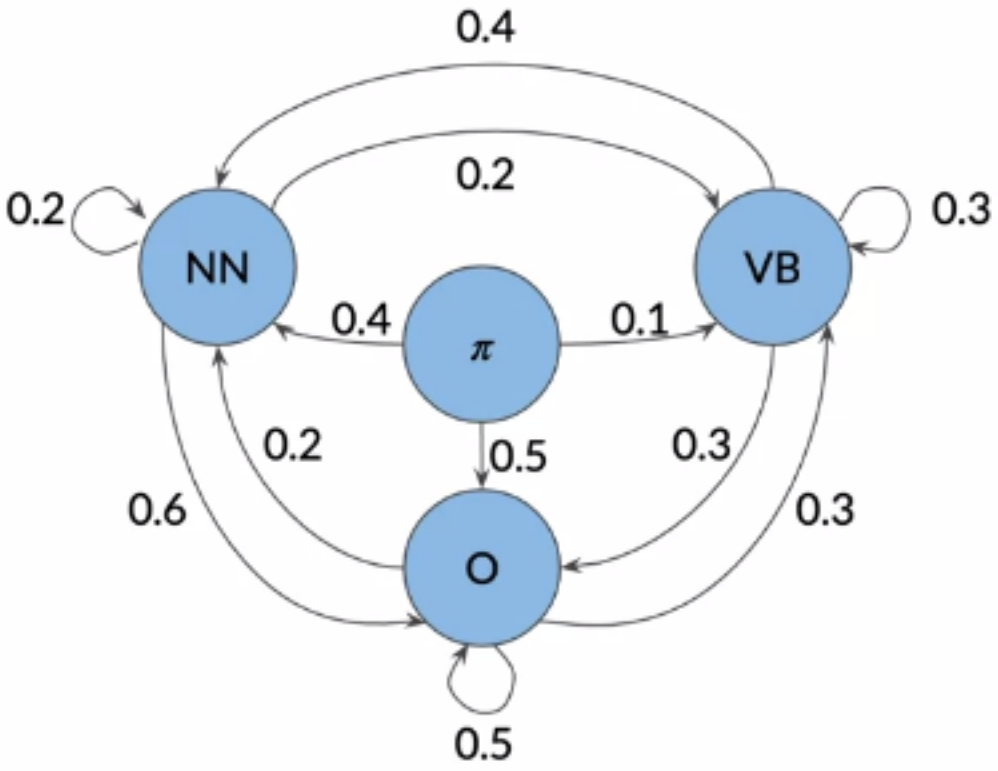
The first word



Why not learn something ?

NN?
VB?
O?

Initial probabilities



$A =$

	NN	VB	O
π (initial)	0.4	0.1	0.5
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

Transition table and matrix

$A =$

	NN	VB	O
π (initial)	0.4	0.1	0.5
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

$$A = \begin{pmatrix} 0.4 & 0.1 & 0.5 \\ 0.2 & 0.2 & 0.6 \\ 0.4 & 0.3 & 0.3 \\ 0.2 & 0.3 & 0.5 \end{pmatrix}$$

Summary

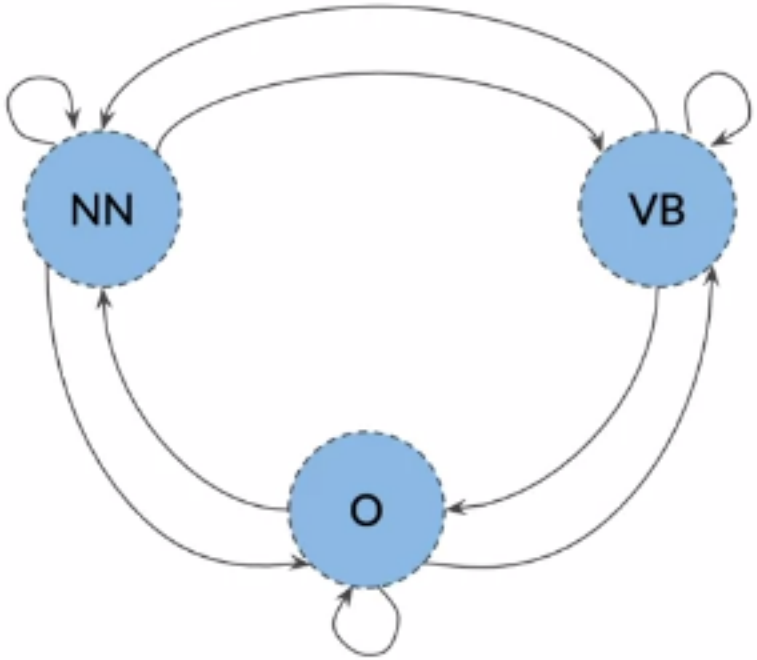
States

$$Q = \{q_1, \ldots, q_N\}$$

Transition matrix

$$A = \begin{pmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{N+1,1} & \cdots & a_{N+1,N} \end{pmatrix}$$

Hidden Markov Model



hidden states

you

jump = verb

you

jump = verb

machine

jump = ?

you

jump = verb
run = verb
fly = verb

machine

jump
run
fly

you

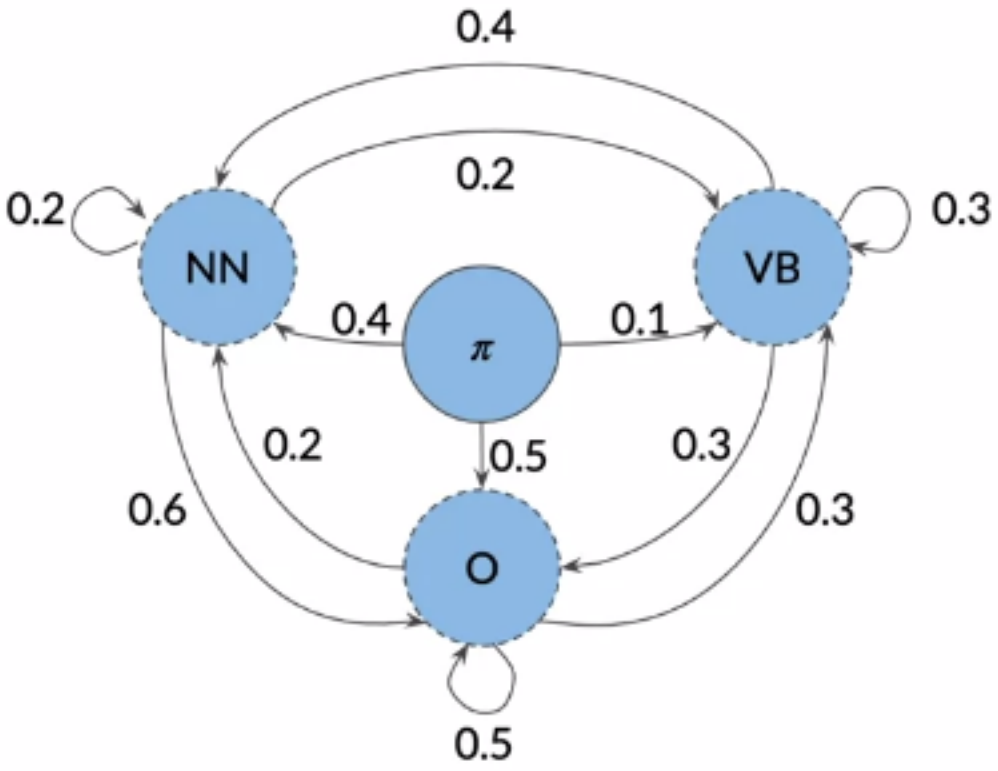
jump = verb
run = verb
fly = verb

machine

jump
run
fly
*

*observable

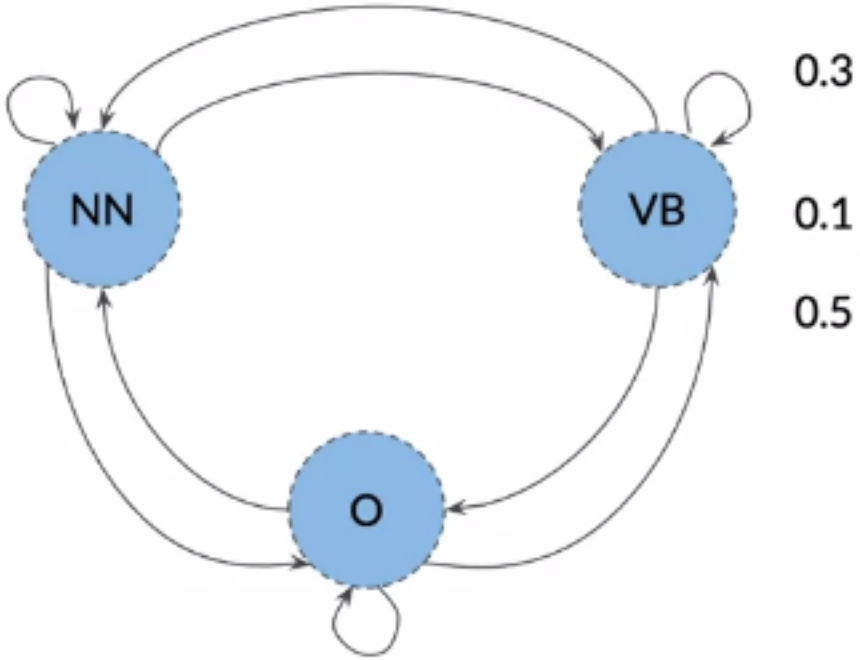
Transition probabilities



$A =$

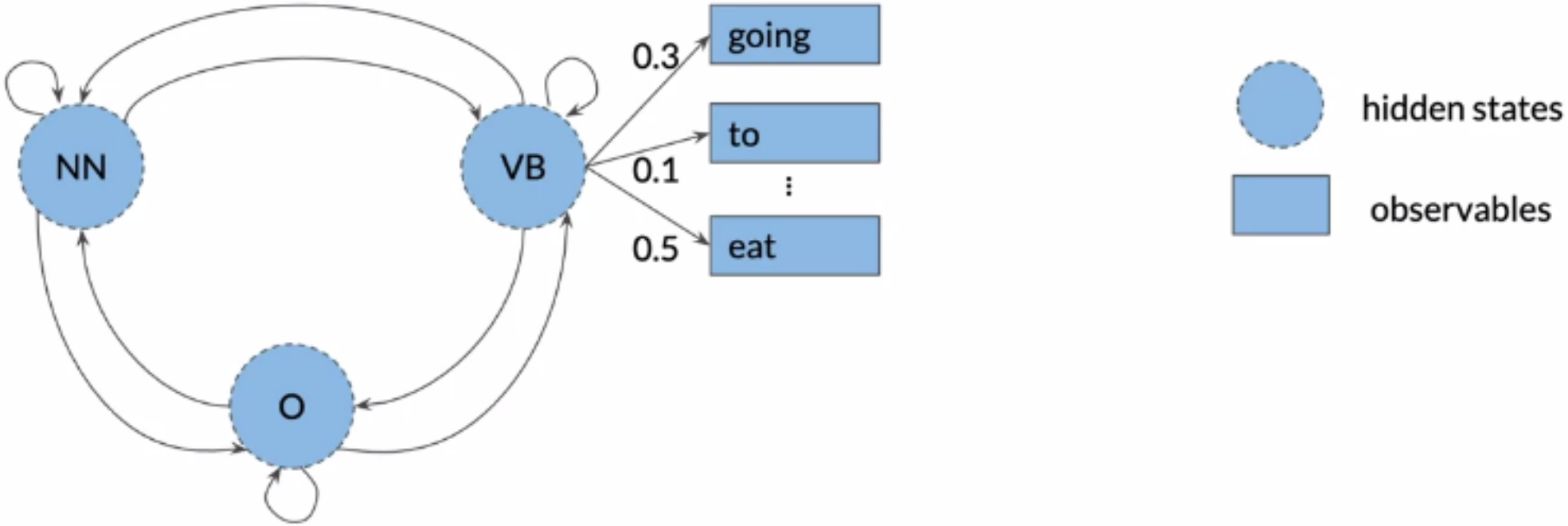
	NN	VB	O
π (initial)	0.4	0.1	0.5
NN (noun)	0.2	0.2	0.6
VB (verb)	0.4	0.3	0.3
O (other)	0.2	0.3	0.5

Emission probabilities

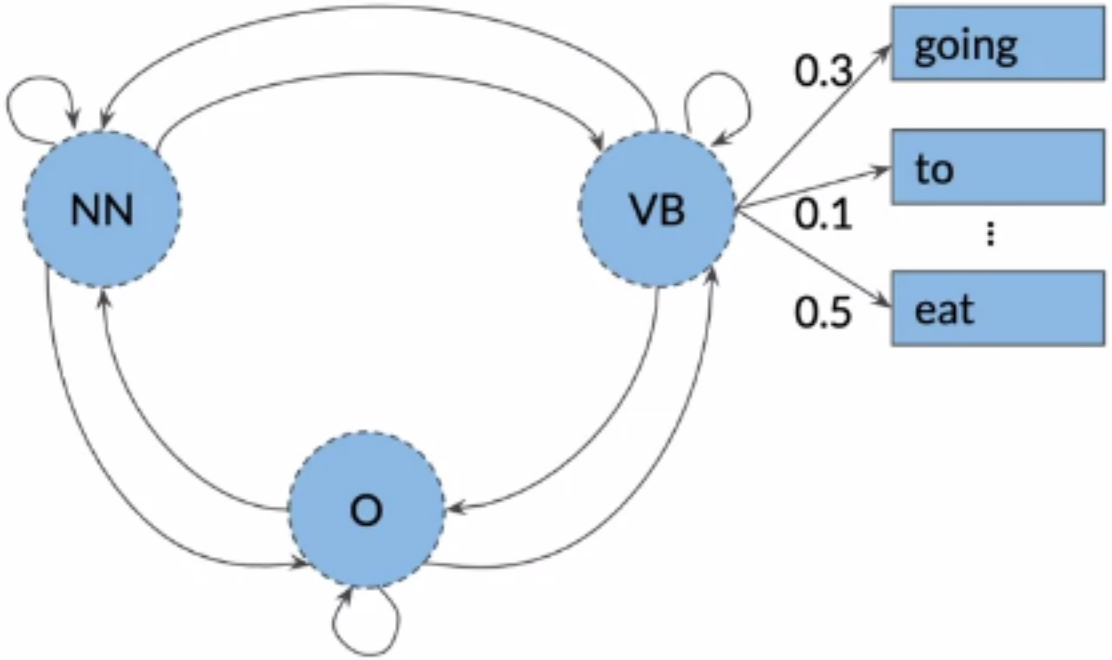


hidden states

Emission probabilities



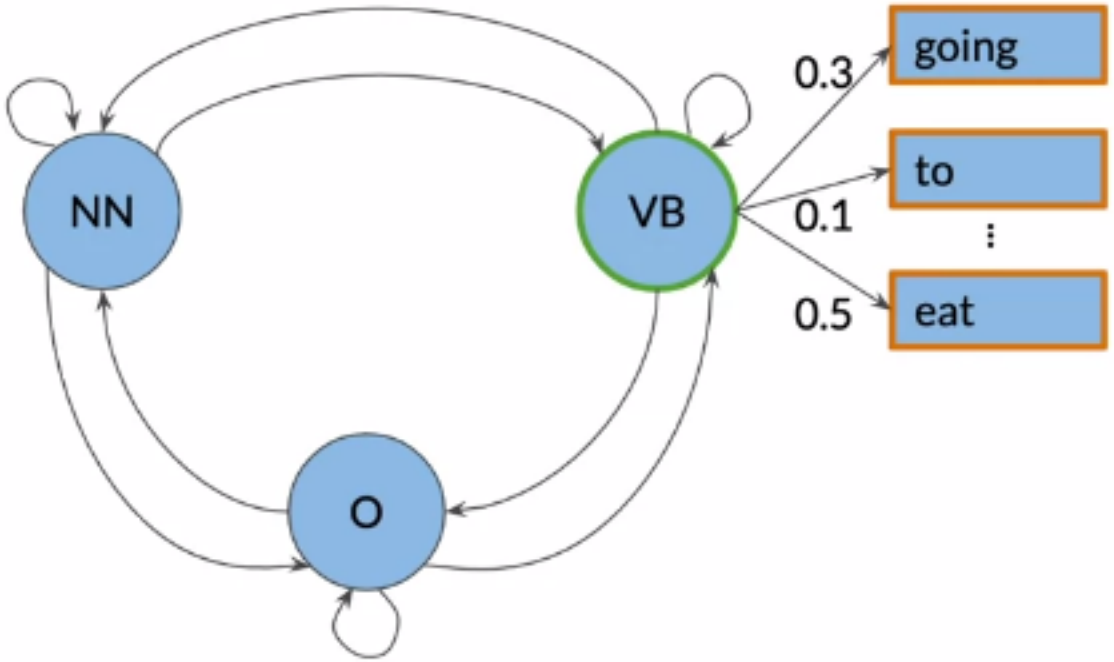
Emission probabilities



$B =$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

Emission probabilities



$B =$

	going	to	eat	...
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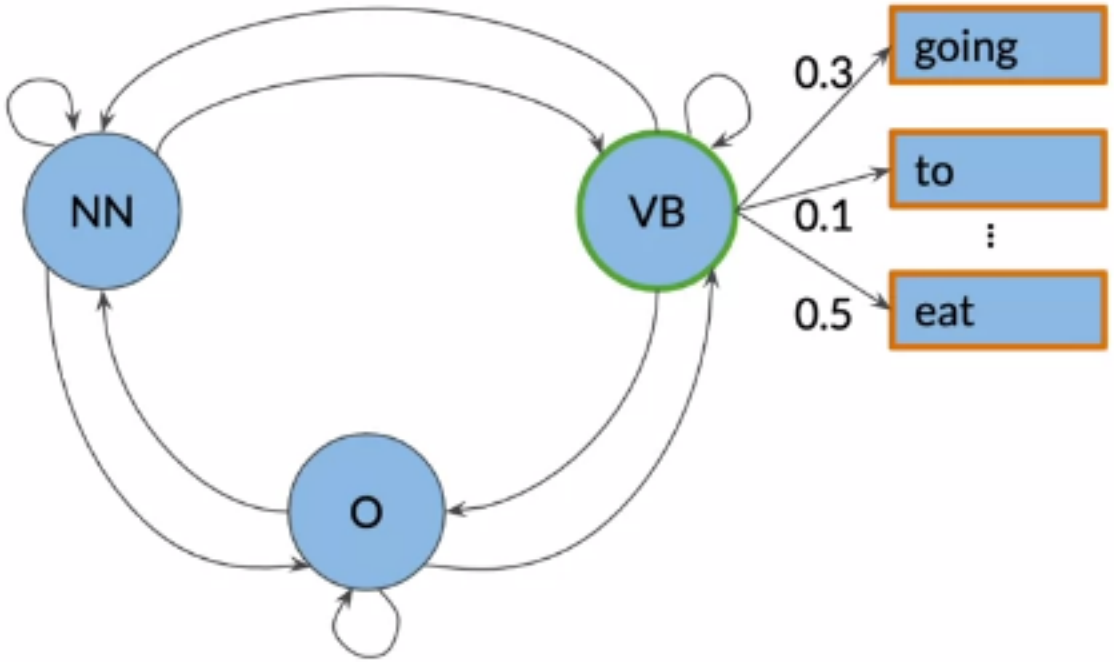
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

Emission probabilities



$B =$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

The emission matrix

$B =$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

$$\sum_{j=1}^{V} b_{ij} = 1$$

The emission matrix

$B =$

	going	to	eat	...
NN (noun)	0.5	0.1	0.02	
VB (verb)	0.3	0.1	0.5	
O (other)	0.3	0.5	0.68	

$$\sum_{j=1}^{V} b_{ij} = 1$$

He lay on his back.

I'll be back.

Summary

States

$$Q = \{q_1, \ldots, q_N\}$$

Transition matrix

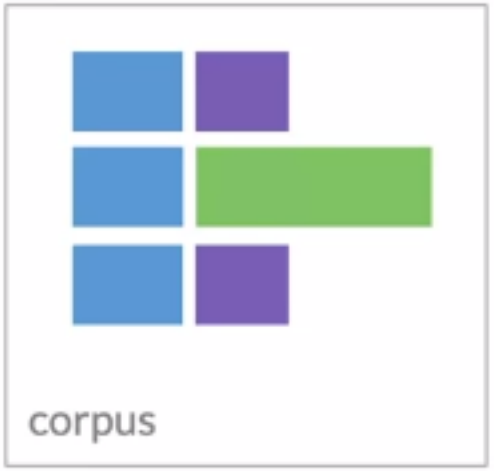
$$A = \begin{pmatrix} a_{1,1} & \cdots & a_{1,N} \\ \vdots & \ddots & \vdots \\ a_{N+1,1} & \cdots & a_{N+1,N} \end{pmatrix}$$

Emission matrix

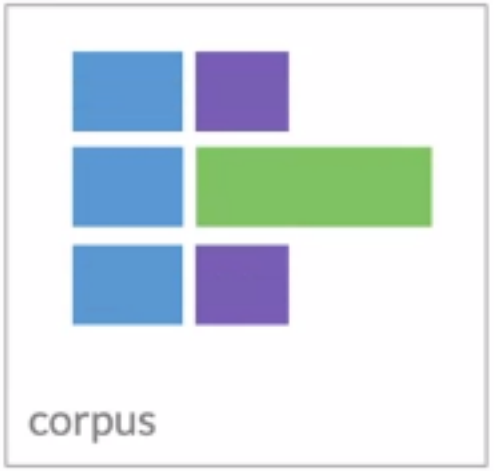
$$B = \begin{pmatrix} b_{11} & \cdots & b_{1V} \\ \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{NV} \end{pmatrix}$$

Transition probabilities

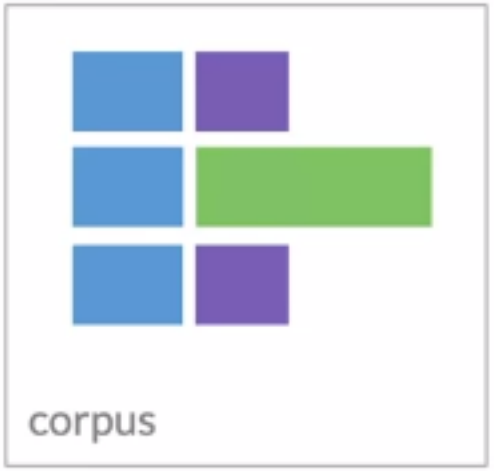
Transition probabilities



Transition probabilities

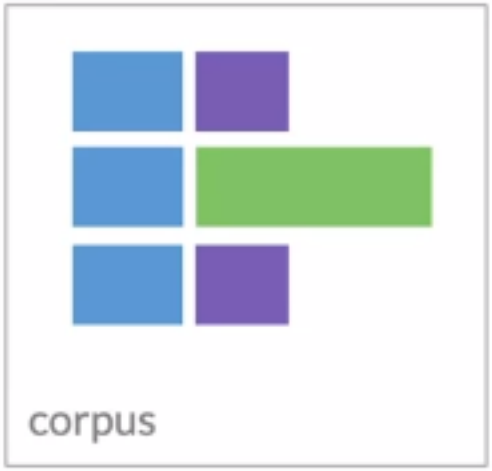


Transition probabilities



Count: 2

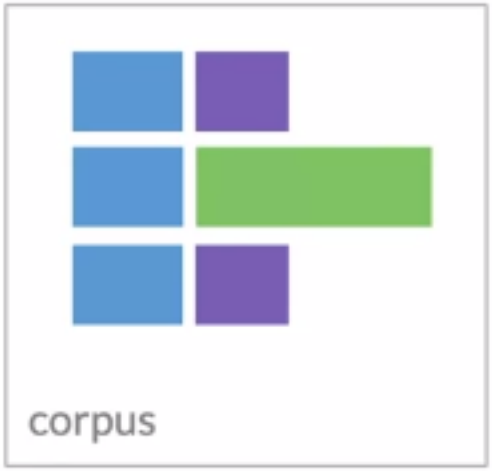
Transition probabilities



Count: 2

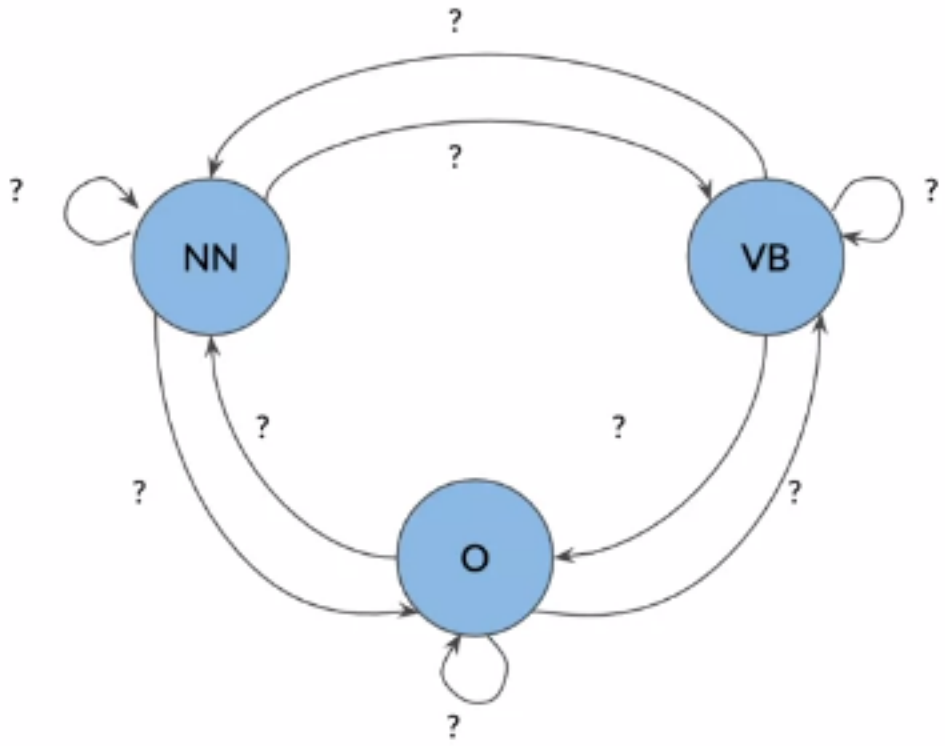
Count: 3

Transition probabilities



transition probability:  +  = ⅔

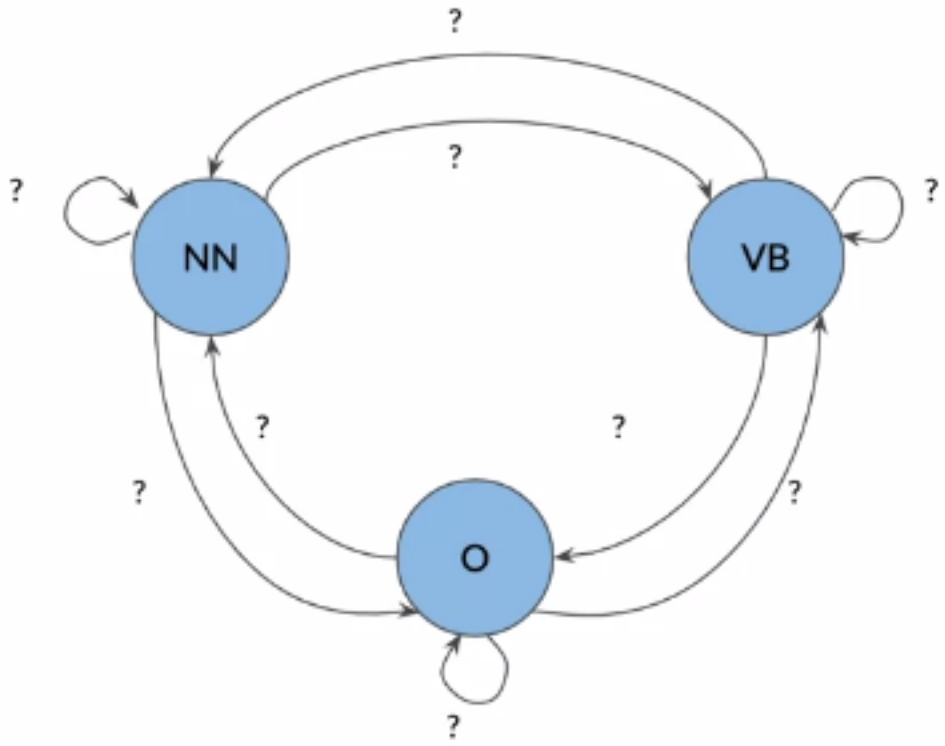
Transition probabilities



1. Count occurrences of tag pairs

$$C(t_{i-1}, t_i)$$

Transition probabilities



1. Count occurrences of tag pairs

$$C(t_{i-1}, t_i)$$

2. Calculate probabilities using the counts

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i)}{\sum_{j=1}^{N} C(t_{i-1}, t_j)}$$

The corpus

In a Station of the Metro

The apparition of these faces in the crowd :

Petals on a wet , black bough .

Ezra Pound – 1913

Preparation of the corpus

<s> In a Station of the Metro

<s> The apparition of these faces in the crowd :

<s> Petals on a wet , black bough .

Ezra Pound – 1913

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Populating the Transition Matrix

Populating the transition matrix

$A =$

	NN	VB	O
π			
NN (noun)			
VB (verb)			
O (other)			

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π			
NN (noun)			
VB (verb)			
O (other)			

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	$C(\pi,\mathrm{NN})$		
NN (noun)	$C(\mathrm{NN},\mathrm{NN})$		
VB (verb)	$C(\mathrm{VB},\mathrm{NN})$		
O (other)	$C(\mathrm{O},\mathrm{NN})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1		
NN (noun)	$C(\text{NN},\text{NN})$		
VB (verb)	$C(\text{VB},\text{NN})$		
O (other)	$C(\text{O},\text{NN})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1		
NN (noun)	0		
VB (verb)	$C(\mathrm{VB},\mathrm{NN})$		
O (other)	$C(\mathrm{O},\mathrm{NN})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1		
NN (noun)	0		
VB (verb)	0		
O (other)	$C(\mathrm{O},\mathrm{NN})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1		
NN (noun)	0		
VB (verb)	0		
O (other)	6		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1	0	
NN (noun)	0	0	
VB (verb)	0	0	0
O (other)	6	0	

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1	0	2
NN (noun)	0	0	
VB (verb)	0	0	0
O (other)	6	0	

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1	0	2
NN (noun)	0	0	6
VB (verb)	0	0	0
O (other)	6	0	

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1	0	2
NN (noun)	0	0	6
VB (verb)	0	0	0
O (other)	6	0	8

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

Populating the transition matrix

$A =$

	NN	VB	O
π	1	0	2
NN	0	0	6
VB	0	0	0
O	6	0	8

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i)}{\sum_{j=1}^{N} C(t_{i-1}, t_j)}$$

Populating the transition matrix

$A =$

	NN	VB	O	
π	1	0	2	3
NN	0	0	6	6
VB	0	0	0	0
O	6	0	8	14

$$P(\mathrm{NN}|\pi) = \frac{C(\pi, \mathrm{NN})}{\sum_{j=1}^{N} C(\pi, t_j)} = \frac{1}{3}$$

Populating the transition matrix

$A =$

	NN	VB	O	
π	1	0	2	3
NN	0	0	6	6
VB	0	0	0	0
O	6	0	8	14

$$P(\mathrm{NN}|\mathrm{O}) = \frac{C(\mathrm{O},\mathrm{NN})}{\sum_{j=1}^{N} C(\mathrm{O},t_j)} = \frac{6}{14}$$

Smoothing

$A =$

	NN	VB	O	
π	1+ε	0+ε	2+ε	3+3*ε
NN	0+ε	0+ε	6+ε	6+3*ε
VB	0+ε	0+ε	0+ε	0+3*ε
O	6+ε	0+ε	8+ε	14+3*ε

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i) + \boxed{\epsilon}}{\sum_{j=1}^{N} C(t_{i-1}, t_j) + \boxed{N} * \boxed{\epsilon}}$$

Smoothing

$A =$

	NN	VB	O
π	0.3333	0.0003	0.6663
NN	0.0001	0.0001	0.9996
VB	0.3333	0.3333	0.3333
O	0.4285	0.0000	0.5713

$$P(t_i|t_{i-1}) = \frac{C(t_{i-1}, t_i) + \epsilon}{\sum_{j=1}^{N} C(t_{i-1}, t_j) + N * \epsilon}$$

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Populating the Emission Matrix

Emission probabilities

Emission probabilities

You

Count: 2

Emission probabilities

You

Count: 2

Count: 3

Emission probabilities

emission probability: You = ⅔

The emission matrix

$B =$

	in	a	...
NN (noun)			
VB (verb)			
O (other)			

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

The emission matrix

$B =$

	in	a	...
NN (noun)	$C(\text{NN}, \text{in})$		
VB (verb)	$C(\text{VB}, \text{in})$		
O (other)	$C(\text{O}, \text{in})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

The emission matrix

$B =$

	in	a	...
NN (noun)	0		
VB (verb)	$C(\text{VB}, \text{in})$		
O (other)	$C(\text{O}, \text{in})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

The emission matrix

$B =$

	in	a	...
NN (noun)	0		
VB (verb)	0		
O (other)	$C(\mathrm{O}, \mathrm{in})$		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

The emission matrix

$B =$

	in	a	...
NN (noun)	0		
VB (verb)	0		
O (other)	2		

<s> in a station of the metro

<s> the apparition of these faces in the crowd :

<s> petals on a wet , black bough .

Ezra Pound – 1913

The emission matrix

$B =$

	in	a	...
NN (noun)	0	...	...
VB (verb)	0	...	...
O (other)	2	...	...

$$P(w_i|t_i) = \frac{C(t_i, w_i) + \epsilon}{\sum_{j=1}^{V} C(t_i, w_j) + N * \epsilon}$$

$$= \frac{C(t_i, w_i) + \epsilon}{C(t_i) + N * \epsilon}$$

Summary

1. Calculate transition and emission matrix

2. How to apply smoothing

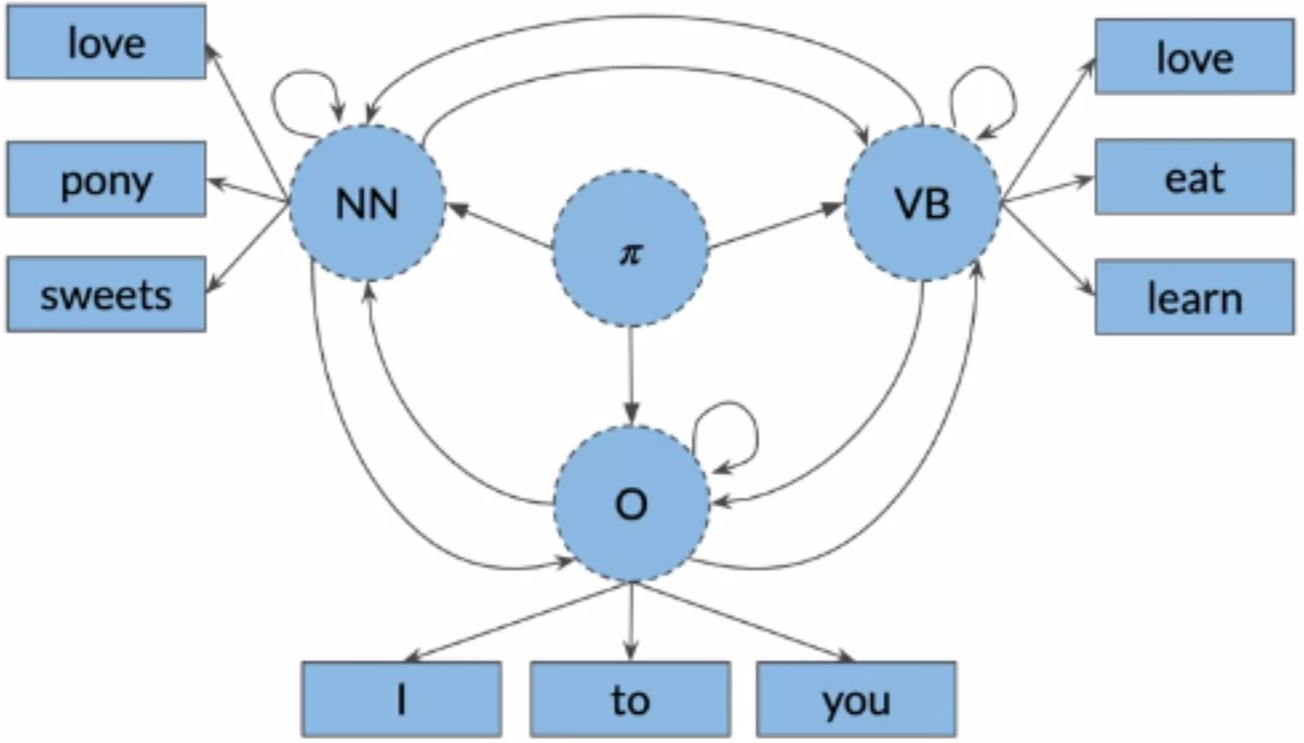
Why not learn something ?

Why not learn something ?

? ? ? ? ?

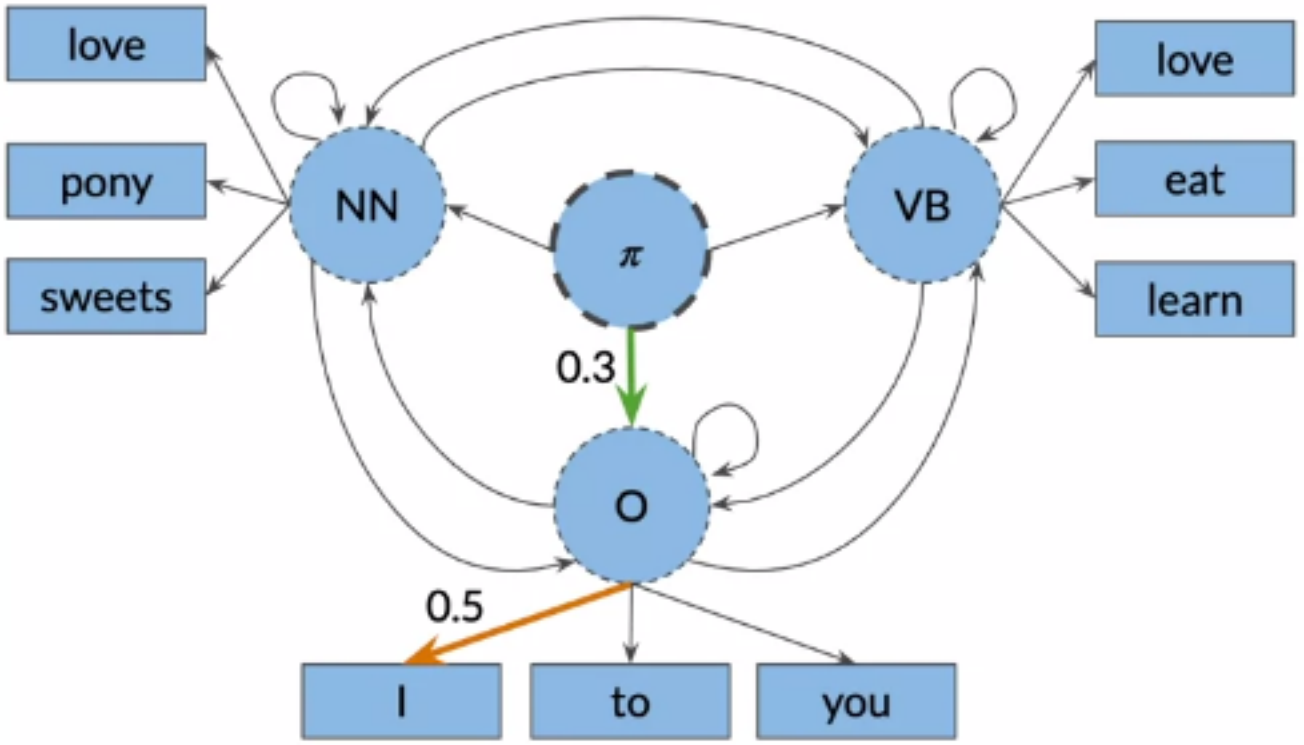
Viterbi algorithm – a graph algorithm

Viterbi algorithm – a graph algorithm



<s> I love to learn

Viterbi algorithm – a graph algorithm

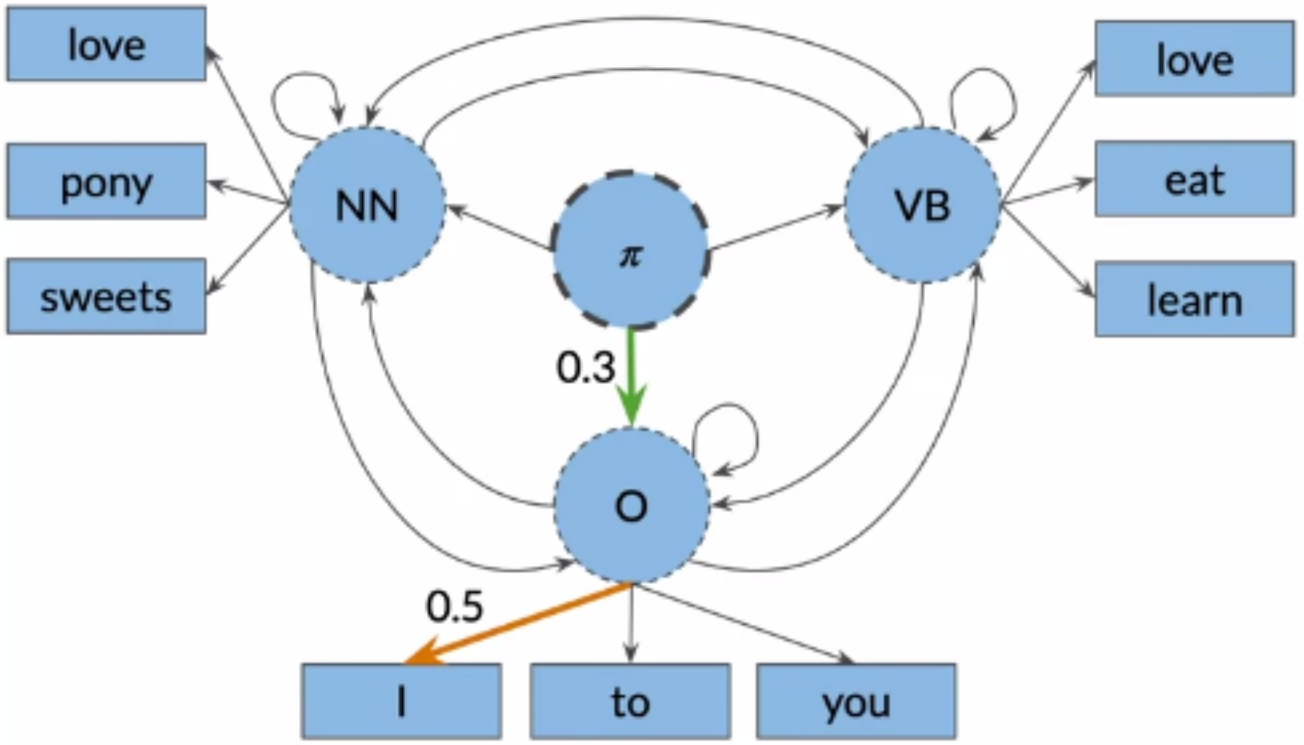


<s> I love to learn

π → O

0.15

Viterbi algorithm – a graph algorithm

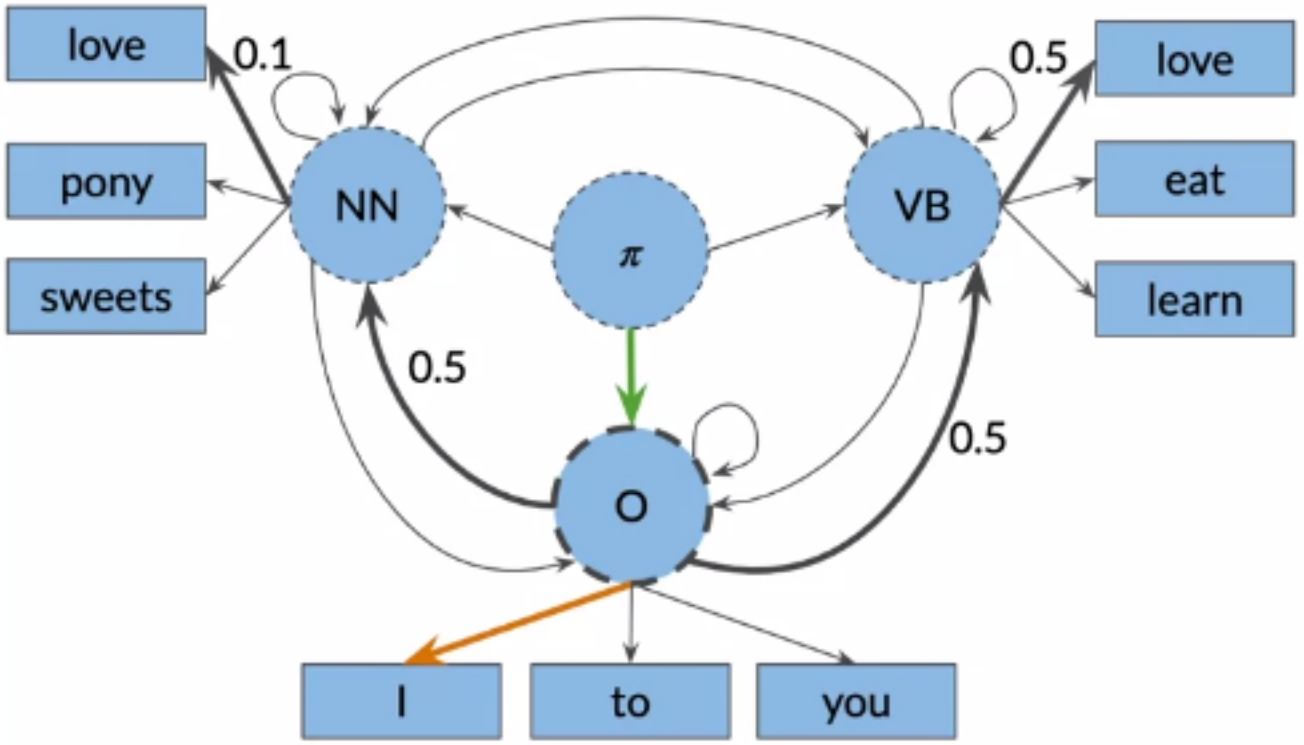


<s> I love to learn

π → O

0.15

Viterbi algorithm – a graph algorithm

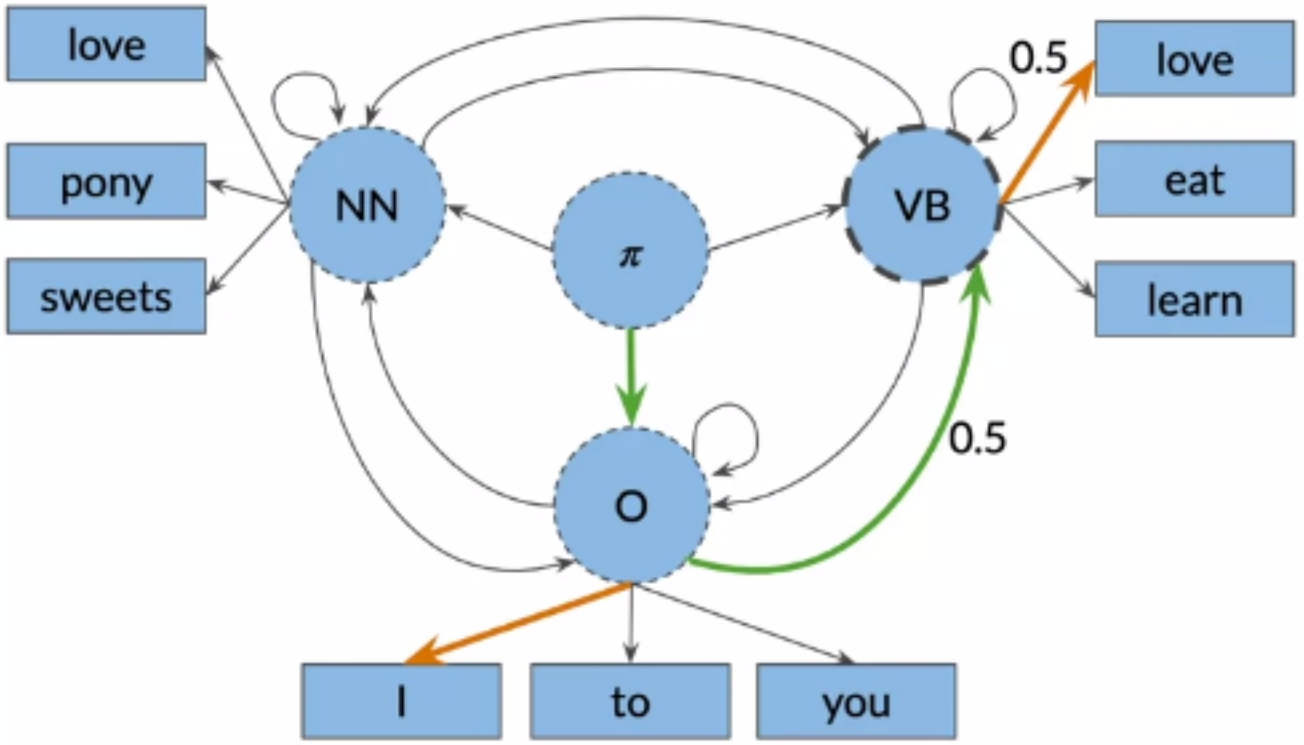


<s> I love to learn

π → O

0.15

Viterbi algorithm – a graph algorithm

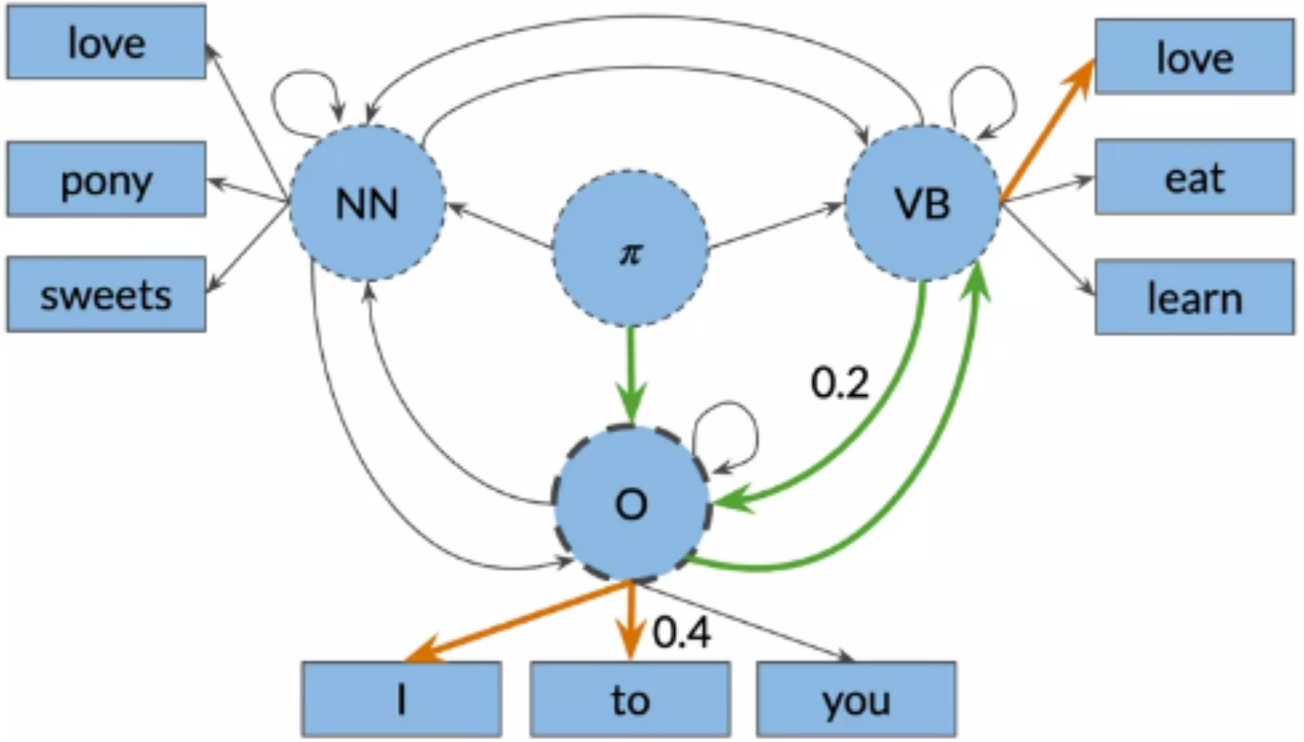


<s> I love to learn

π → O → VB

0.15 0.25

Viterbi algorithm – a graph algorithm

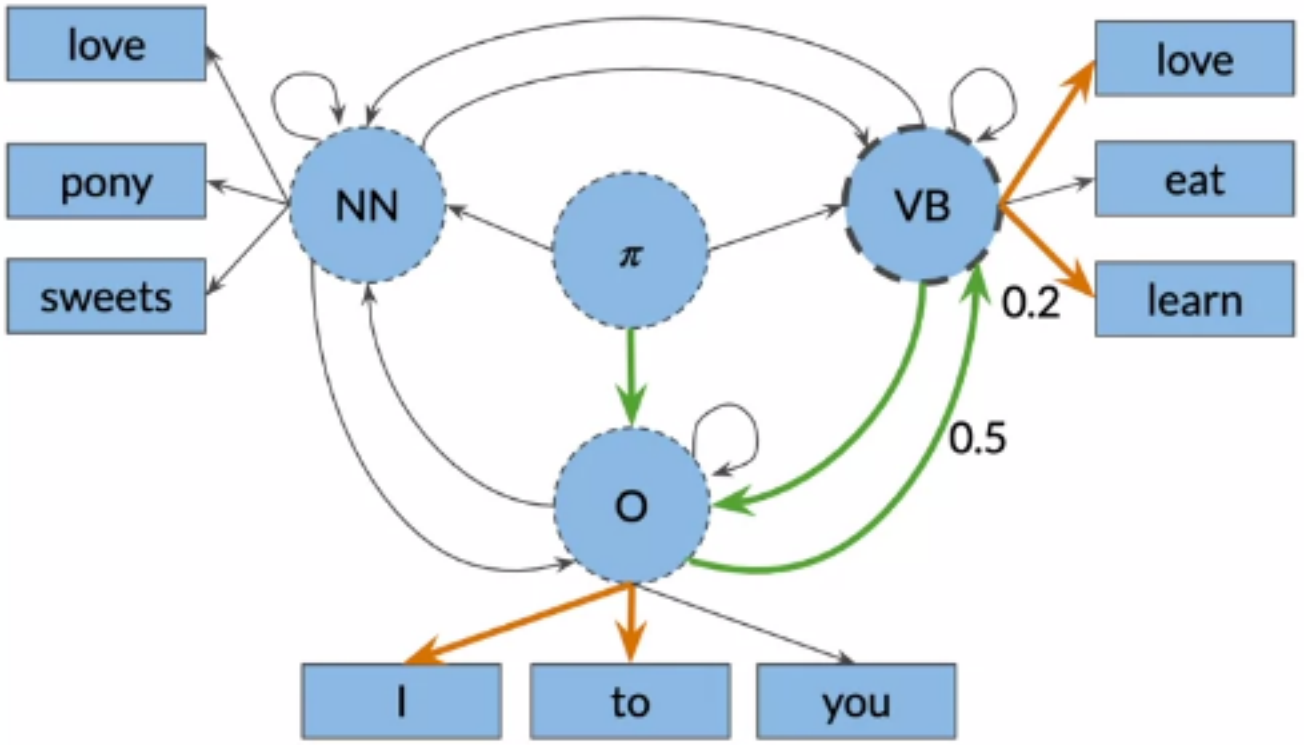


<s> I love to learn

π → O → VB → O

0.15 0.25 0.08

Viterbi algorithm – a graph algorithm

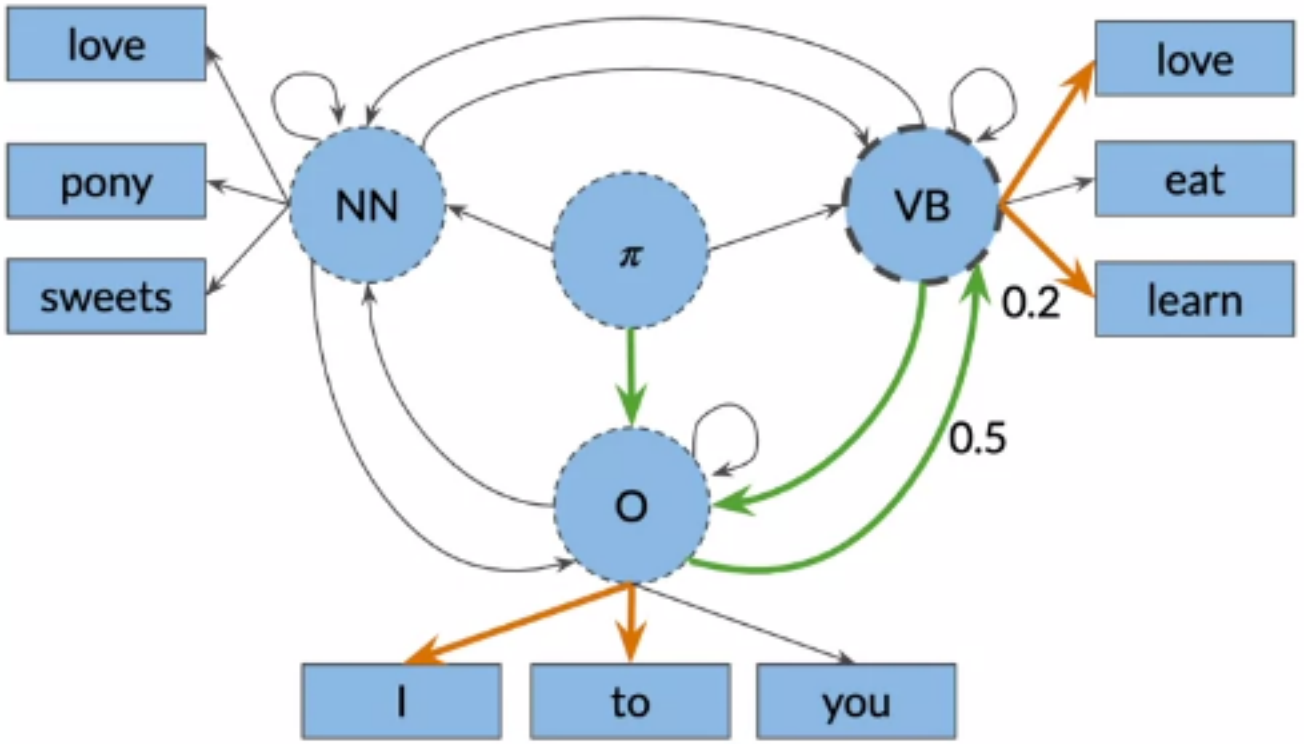


<s> I love to learn

π → O → VB → O → VB

0.15 0.25 0.08 0.1

Viterbi algorithm – a graph algorithm



<s> I love to learn

π → O → VB → O → VB

0.15 * 0.25 * 0.08 * 0.1

Probability for this sequence of hidden states: 0.0003

Viterbi algorithm – Steps

1. Initialization step
2. Forward pass
3. Backward pass

Viterbi algorithm – Steps

1. Initialization step
2. Forward pass
3. Backward pass

$C =$

	w_1	w_2	...	w_K
t_1				
...				
t_N				

$D =$

	w_1	w_2	...	w_K
t_1				
...				
t_N				

Viterbi algorithm – Steps

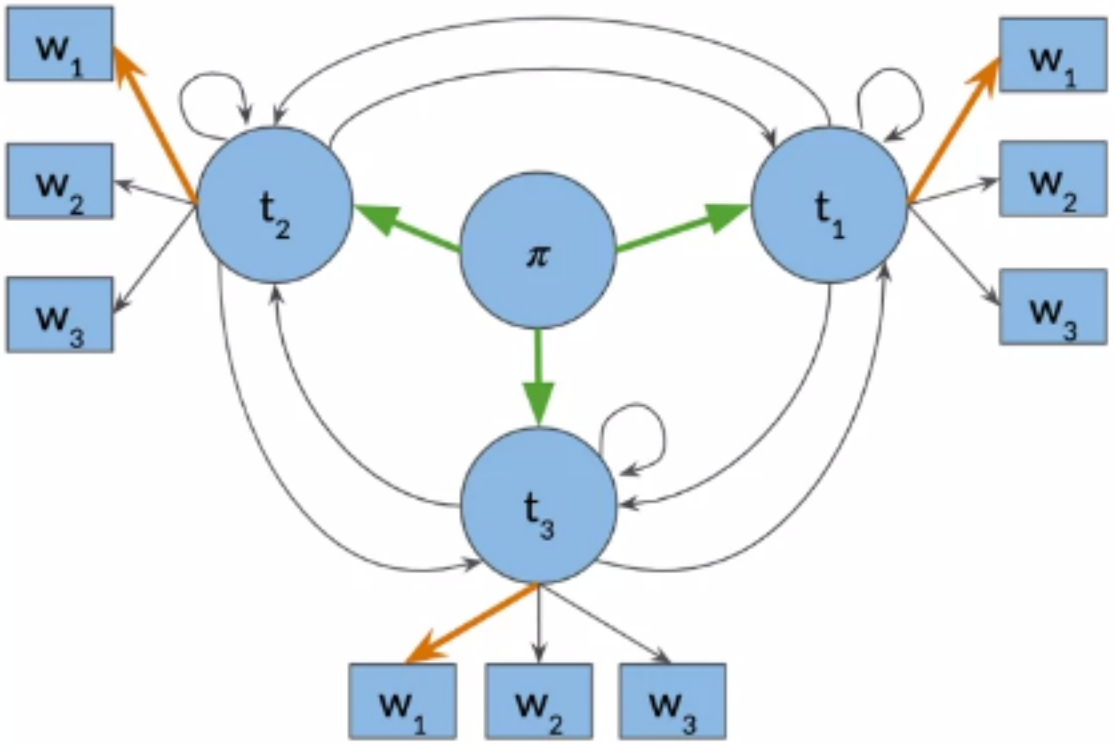
1. Initialization step

Initialization step

$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$			
...				
t_N	$c_{N,1}$			

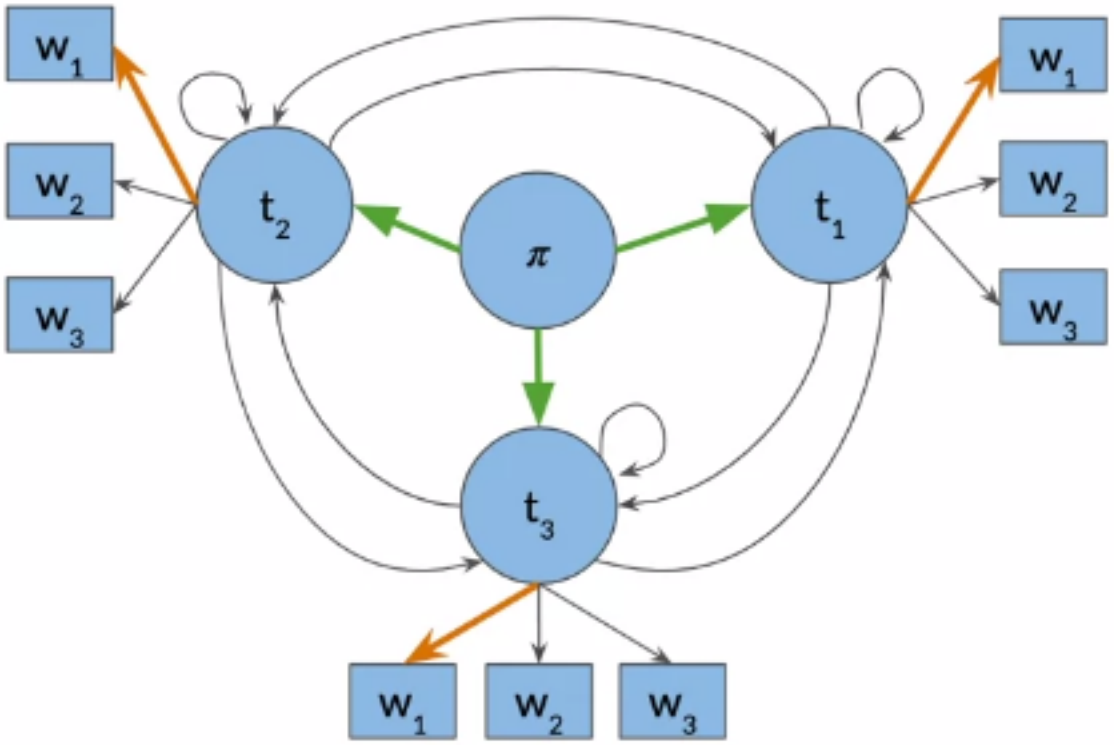
Initialization step



$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$			
...				
t_N	$c_{N,1}$			

Initialization step

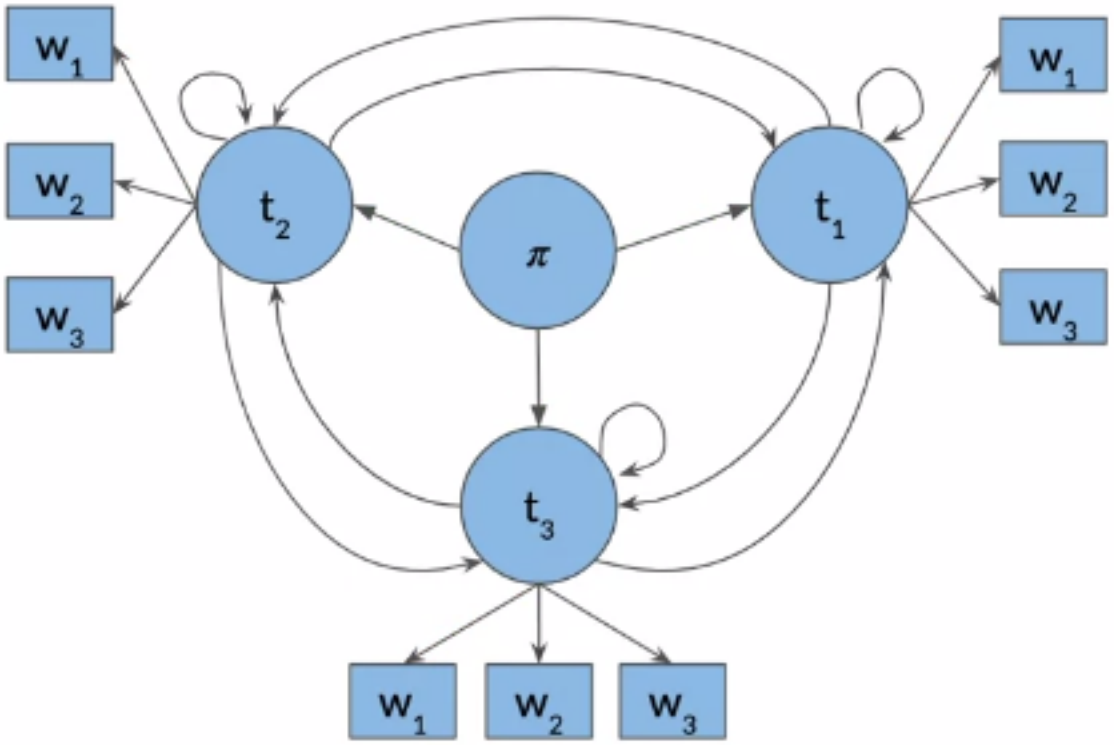


$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$			
...				
t_N	$c_{N,1}$			

$$c_{i,1} = \pi_i * b_{i,cindex(w_1)}$$
$$= a_{1,i} * b_{i,cindex(w_1)}$$

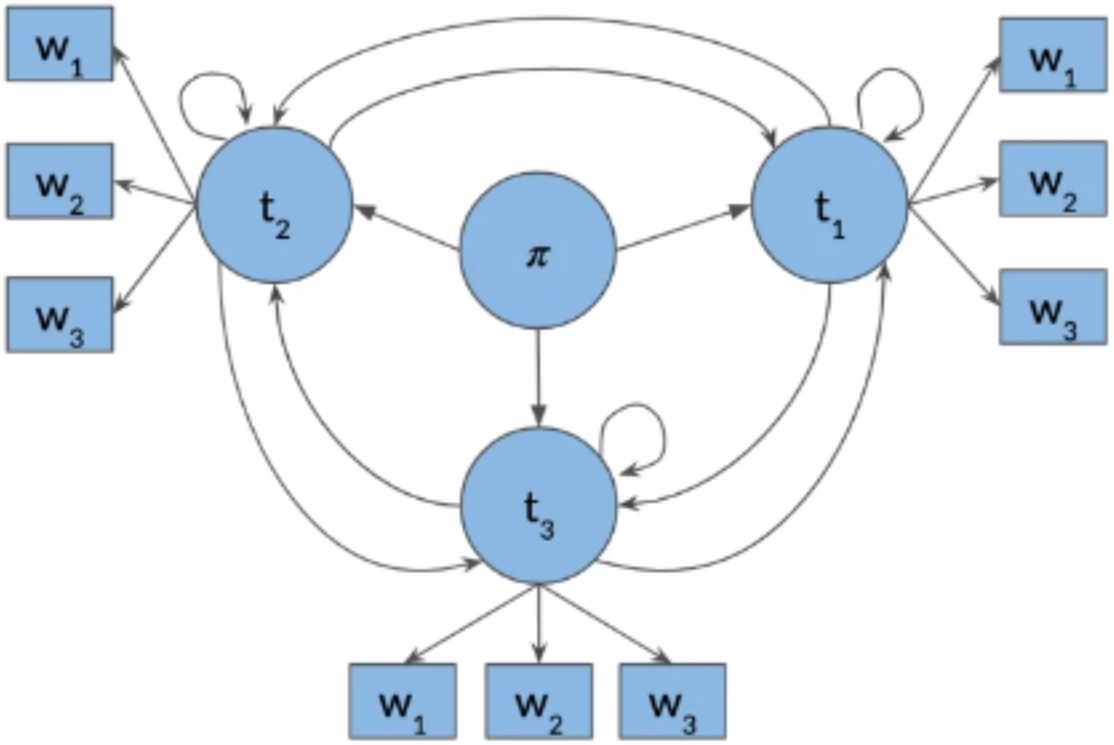
Initialization step



$D =$

	w_1	w_2	...	w_K
t_1	$d_{1,1}$			
...				
t_N	$d_{N,1}$			

Initialization step



$D =$

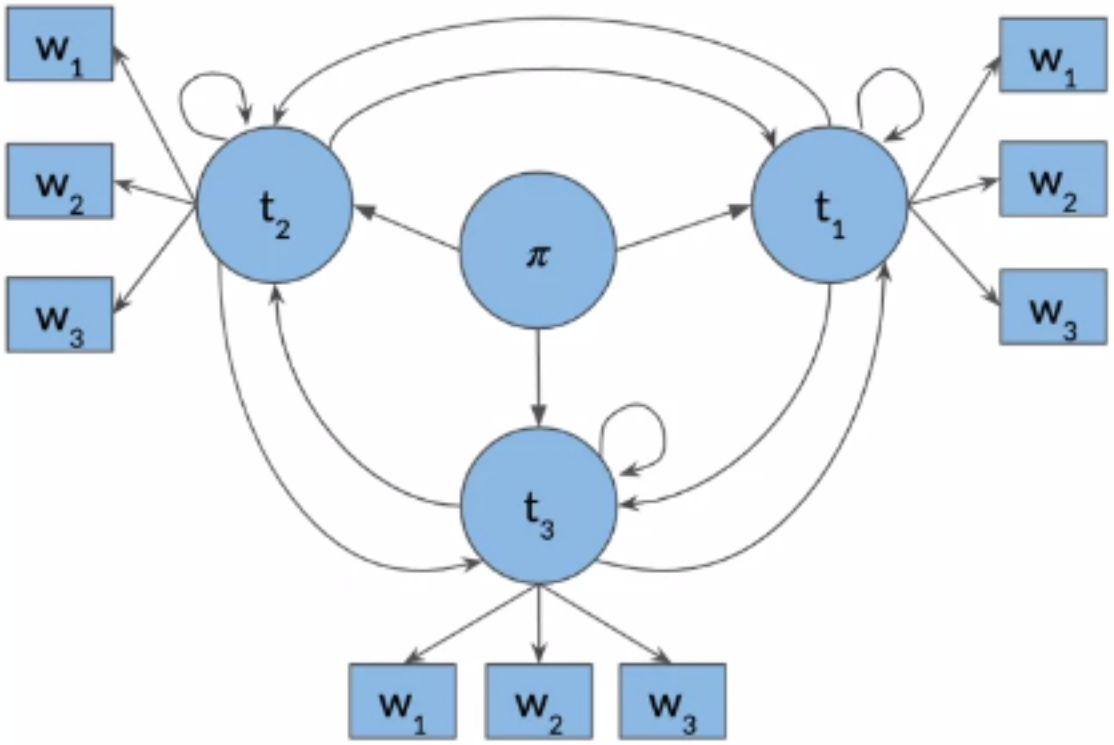
	w_1	w_2	...	w_K
t_1	$d_{1,1}$			
...				
t_N	$d_{N,1}$			

$$d_{i,1} = 0$$

Viterbi algorithm – Steps

2. Forward pass

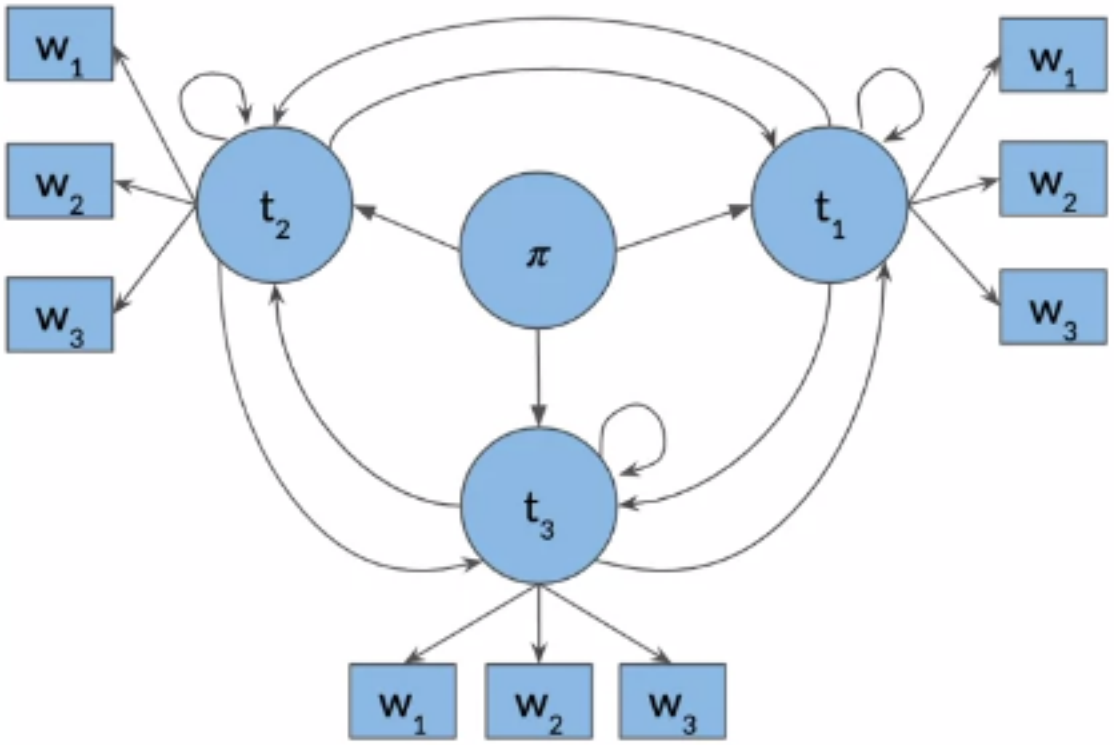
Forward pass



$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

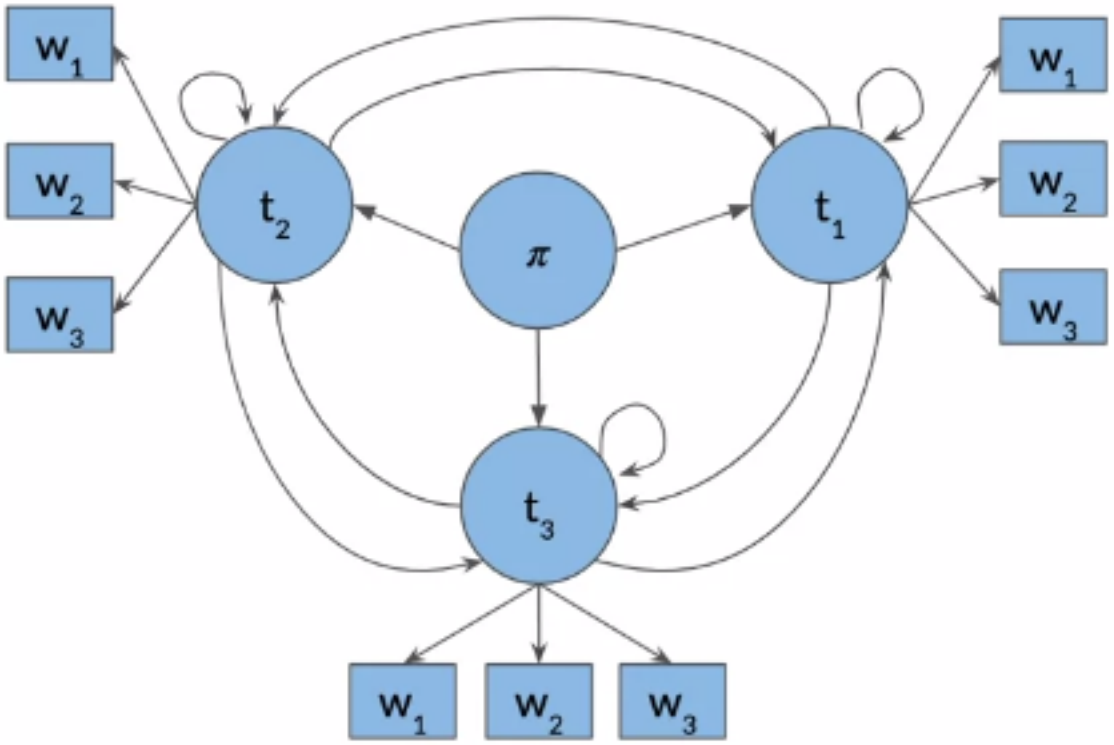
Forward pass



$$C = \begin{array}{|c|c|c|c|c|} \hline & w_1 & w_2 & \dots & w_K \\ \hline t_1 & c_{1,1} & c_{1,2} & & c_{1,K} \\ \hline \dots & & & & \\ \hline t_N & c_{N,1} & c_{N,2} & & c_{N,K} \\ \hline \end{array}$$

$$c_{i,j} = \max_k c_{k,j-1} * a_{k,i} * b_{i,cindex(w_j)}$$

Forward pass

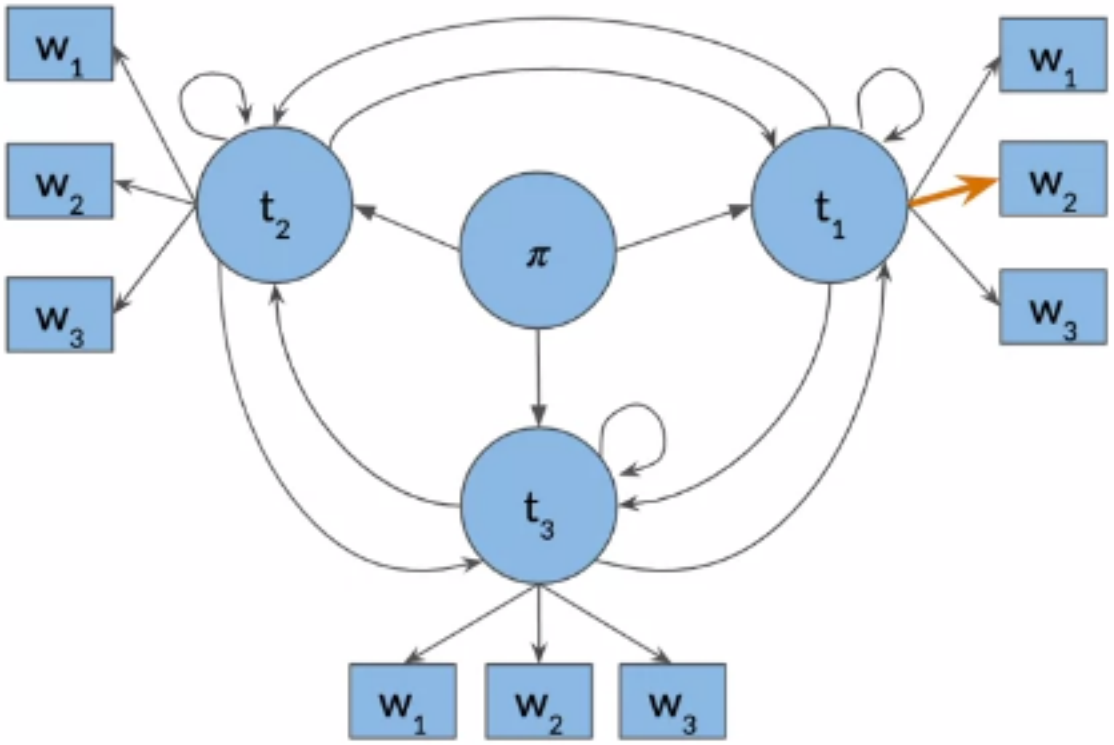


$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$$c_{1,2} = \max_k c_{k,1} * a_{k,1} * b_{1,cindex(w_2)}$$

Forward pass

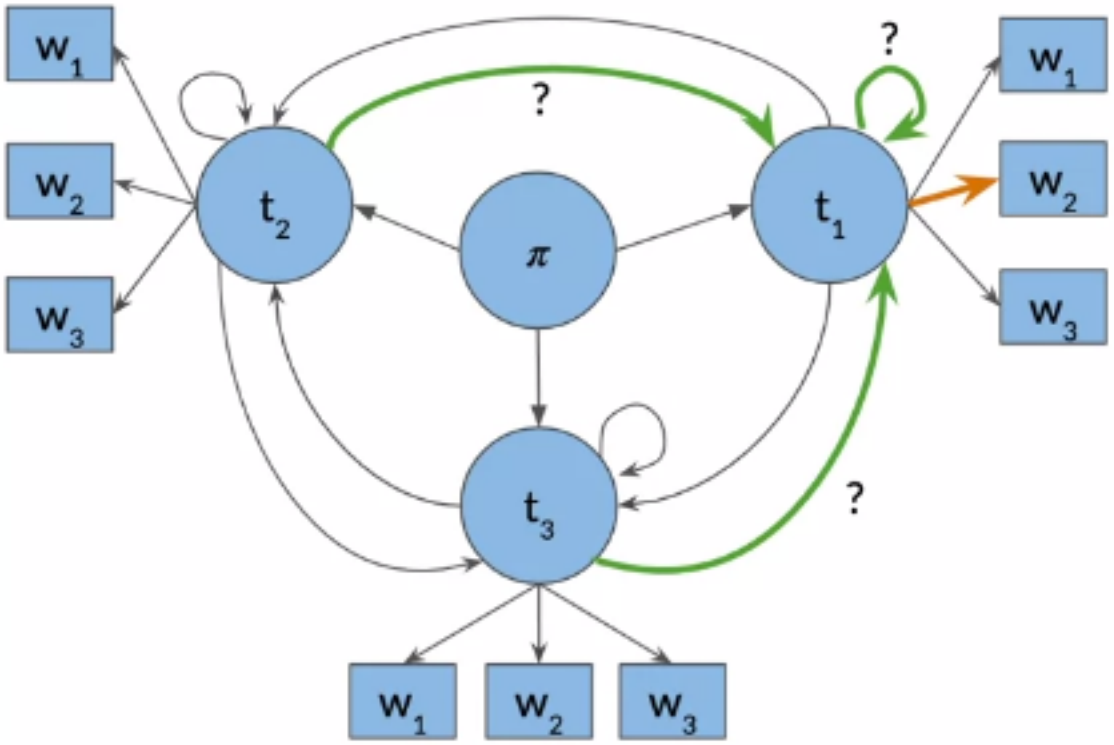


$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$$c_{1,2} = \max_k c_{k,1} * a_{k,1} * b_{1,cindex(w_2)}$$

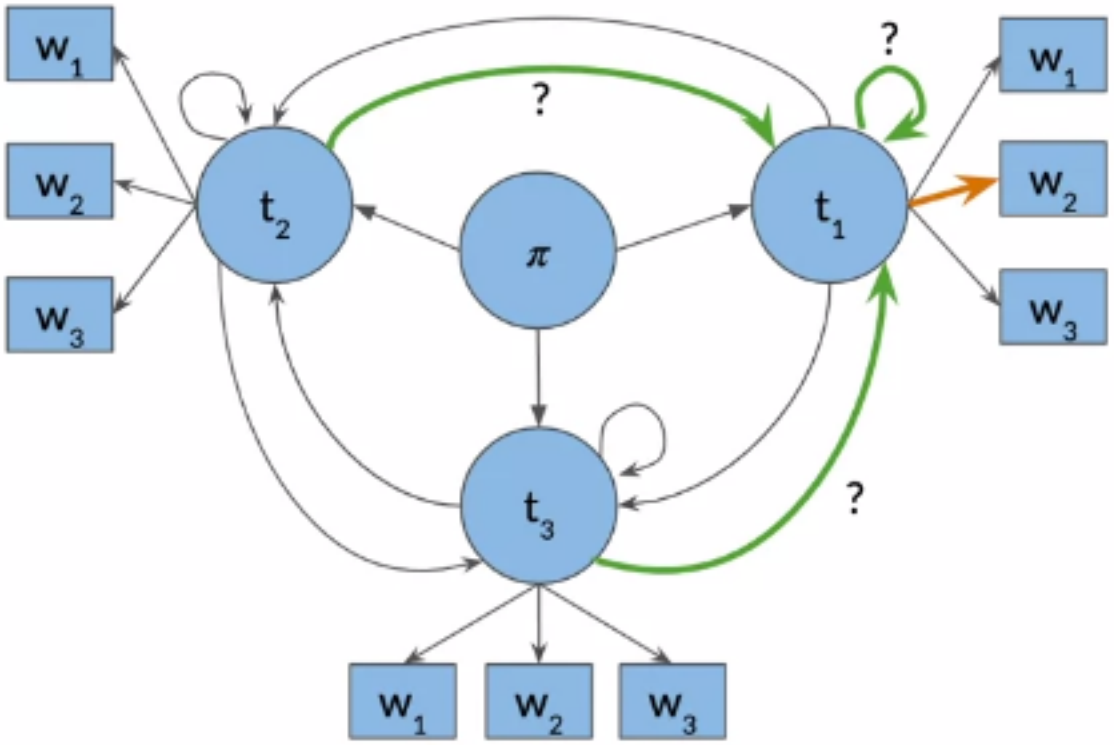
Forward pass



$$C = \begin{array}{|c|c|c|c|c|} \hline & w_1 & w_2 & \dots & w_K \\ \hline t_1 & c_{1,1} & c_{1,2} & & c_{1,K} \\ \hline \dots & & & & \\ \hline t_N & c_{N,1} & c_{N,2} & & c_{N,K} \\ \hline \end{array}$$

$$c_{1,2} = \max_k c_{k,1} * a_{k,1} * b_{1,cindex(w_2)}$$

Forward pass

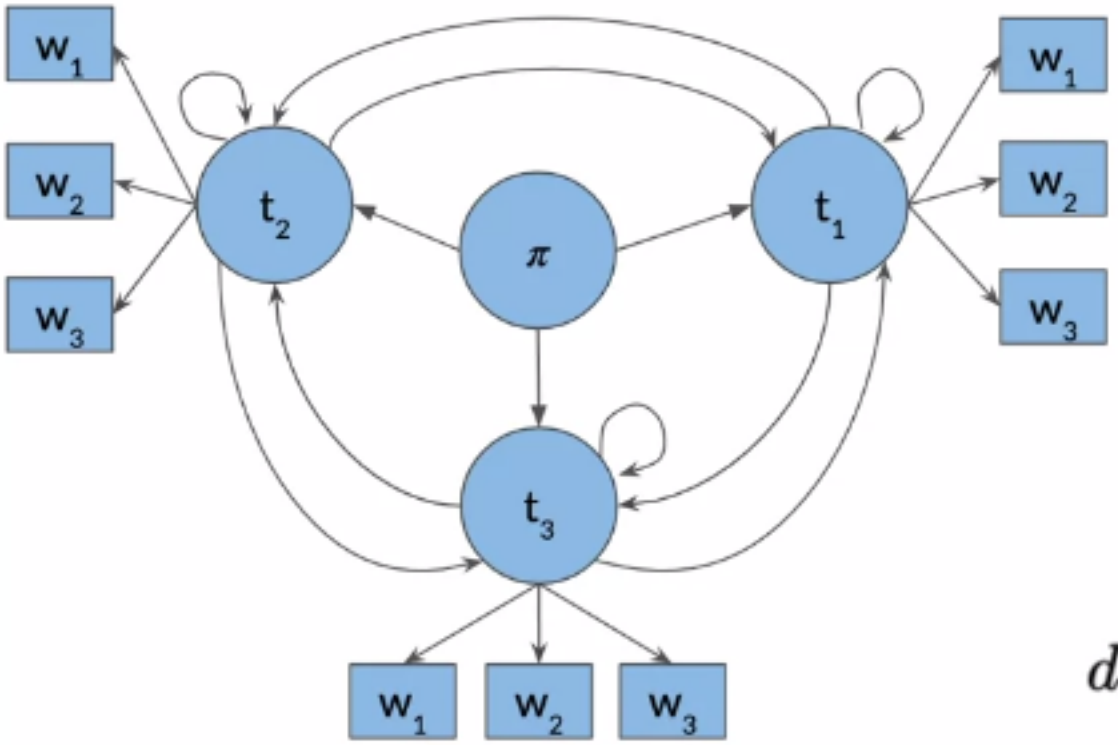


$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$$c_{1,2} = \max_k c_{k,1} * a_{k,1} * b_{1,cindex(w_2)}$$

Forward pass



$D =$

	w_1	w_2	...	w_K
t_1	$d_{1,1}$	$d_{1,2}$		$d_{1,K}$
...				
t_N	$d_{N,1}$	$d_{N,2}$		$d_{N,K}$

$$c_{i,j} = \max_k c_{k,j-1} * a_{k,i} * b_{i,cindex(w_j)}$$

$$d_{i,j} = \operatorname{argmax}_k c_{k,j-1} * a_{k,i} * b_{i,cindex(w_j)}$$

Viterbi algorithm – Steps

3. Backward pass

Backward pass

$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$D =$

	w_1	w_2	...	w_K
t_1	$d_{1,1}$	$d_{1,2}$		$d_{1,K}$
...				
t_N	$d_{N,1}$	$d_{N,2}$		$d_{N,K}$

$$s = \underset{i}{\operatorname{argmax}}\, c_{i,K}$$

Backward pass

$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$D =$

	w_1	w_2	...	w_K
t_1	$d_{1,1}$	$d_{1,2}$		$d_{1,K}$
...				
t_N	$d_{N,1}$	$d_{N,2}$		$d_{N,K}$

$$s = \underset{i}{\operatorname{argmax}}\, c_{i,K}$$

Backward pass

$C =$

	w_1	w_2	...	w_K
t_1	$c_{1,1}$	$c_{1,2}$		$c_{1,K}$
...				
t_N	$c_{N,1}$	$c_{N,2}$		$c_{N,K}$

$$s = \underset{i}{\operatorname{argmax}}\, c_{i,K}$$

$D =$

	w_1	w_2	...	w_K
t_1	$d_{1,1}$	$d_{1,2}$		$d_{1,K}$
...				
t_N	$d_{N,1}$	$d_{N,2}$		$d_{N,K}$

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	1
t_2	0	2	4	2	3
t_3	0	2	4	4	4
t_4	0	4	4	3	1

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	1
t_2	0	2	4	2	3
t_3	0	2	4	4	4
t_4	0	4	4	3	1

<s> w1 w2 w3 w4 w5

Backward pass

$C =$

	w_1	w_2	w_3	w_4	w_5
t_1	0.25	0.125	0.025	0.0125	0.01
t_2	0.1	0.025	0.05	0.01	0.003
t_3	0.3	0.05	0.025	0.02	0.0000
t_4	0.2	0.1	0.000	0.0025	0.0003

$$s = \underset{i}{\operatorname{argmax}}\, c_{i,K} = 1$$

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

$$s = \underset{i}{\operatorname{argmax}}\, c_{i,K} = 1$$

<s> w1 w2 w3 w4 w5

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

<s>	w1	w2	w3	w4	w5
					t_1

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

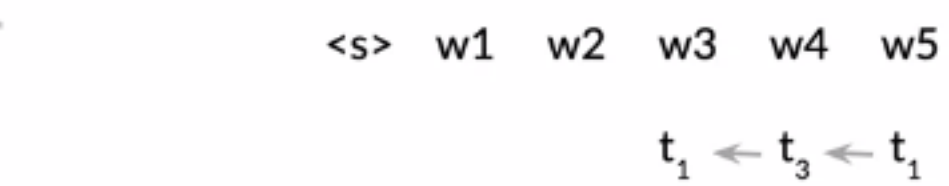
<s> w1 w2 w3 w4 w5

$t_3 \leftarrow t_1$

Backward pass

$D =$

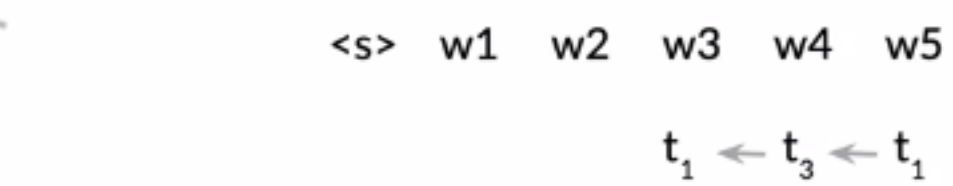
	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1



Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1



Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

<s> w1 w2 w3 w4 w5

$t_1 \leftarrow t_3 \leftarrow t_1$

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

<s> w1 w2 w3 w4 w5

$t_3 \leftarrow t_1 \leftarrow t_3 \leftarrow t_1$

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

<s> w1 w2 w3 w4 w5

$t_3 \leftarrow t_1 \leftarrow t_3 \leftarrow t_1$

Backward pass

$D =$

	w_1	w_2	w_3	w_4	w_5
t_1	0	1	3	2	3
t_2	0	2	4	1	3
t_3	0	2	4	1	4
t_4	0	4	4	3	1

<s>	w1	w2	w3	w4	w5
π	$\leftarrow t_2$	$\leftarrow t_3$	$\leftarrow t_1$	$\leftarrow t_3$	$\leftarrow t_1$

Implementation notes

Implementation notes

1. In Python index starts with 0!
2. Use log probabilities

$$c_{i,j} = \max_k c_{k,j-1} * a_{k,i} * b_{i,cindex(w_j)}$$

$$\downarrow$$

$$log(c_{i,j}) = \max_k log(c_{k,j-1}) + log(a_{k,i}) + log(b_{i,cindex(w_j)})$$